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IMPEDANCE MATCHING FOR GRASPING WITH MECHANICAL FINGERS

By:

Shahram Payandeh

A thesis submitted in conformity with the requirements for the degree of

Doctor of Philosophy

in the University of Toronto

Department of Mechanical Engineering
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Shahram Payandeh

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Room 111, 63 St. George Street

Impedance Matching for Grasping with Mechanical Fingers

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IMPEDANCE MATCHING FOR GRASPING WITH DEXTEROUS MECHANICAL HANDS

By:

Shahram Payandeh

A thesis submitted in confirmity with
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ABSTRACT

The objectives of this research are: i) to model the grasp between the fingers of a dexterous mechanical hand with soft finger-tips and an object; ii) to devise a control law such that the actual linear impedance model of each grasping finger is *matched* with the desired impedance; iii) to develop a control law such that the contact wrenches between each grasping finger and the object are maintained during a task and the presence of uncertainties.

Using the *causality* principle, each grasping finger is modelled as a system consisting of a mass, a spring and a damper in each direction of the end-point reference coordinate frame. In this model, the spring and damper are connected in parallel between the mass model of the finger and the palm of the hand. Similarly, the soft finger-tip is modelled as a spring and damper system connected in parallel between the mass model of the finger and the grasped object. The model of the finger, with its mass, spring and damper parameters given, is referred to as the *targeted impedance*.

In general, the actual linear model of each grasping finger is different from the model specified by a targeted impedance. In order for each finger to have the impedance model specified by the targeted impedance, the concept of impedance matching and a method of implementation are proposed. Implementation of the impedance matching concept is based on the linear dynamic decoupling approach. It is shown that when the exact model parameters of the finger are known, the implementation of this method results in the actual impedance model of the finger to be replaced with the targeted impedance.

When fingers of a dexterous mechanical hand are grasping an object, they must exert wrenches equal to the desired grasping wrenches. Furthermore they must maintain these wrenches during the task and presence of uncertainties. Using the matched impedance model of the finger and the model of the soft finger-tip, a grasping wrench control architecture is developed. This architecture is obtained based on the input/output relationships of the impedance/admittance of these models. It is shown that in the presence of uncertainties in the actual parameters of the finger and the presence of disturbance wrench, the grasping wrench controller would not have the desired performance. Based on the *theory of the servomechanism problem*, the feedforward control impedance/admittance blocks of the general control architecture are modified such that the controller exhibits robustness. The modified architecture has the property that the error between the actual and desired grasping wrenches approaches zero asymptotically. This asymptotic regulation occurs even when there are uncertainties in the parameters of the actual finger, and where there is a constant disturbance wrench. The performance of the robust controller is demonstrated through simulation and shown experimentally using a *2DOF* planar finger with soft finger-tip making contact with a rigid wall. Also, based on the robustness theory, a robust grasping wrench controller is proposed for tooling tasks. In this controller, a model of the *exogenous* disturbance wrench which arises from the interaction of the tool with the environment is included in the control architecture.

RESUME

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S. Payandeh, A. A. Goldenberg, *Formulation of the Kinematic Model of a General (6DOF) Robot Manipulator Using a Screw Operator,* Journal of Robotic Systems, Vol. 4, No. 6, 1987, pp. 771-797

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Conference Publications

H. A. ElMaraghy, **S. Payandeh,** *An Application of Screw Theory to the Robot Manipulator Inverse Kinematics Problem,* Proceedings of Fourth Cairo University Conference, 1988, Cairo

S. Payandeh, A. A. Goldenberg, *A knowledge-Based Approach to Grasping,* Proceedings of IEEE IECON Conference, 1988, Singapore

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H. A. ElMaraghy, **S. Payandeh,** *Contact Reasoning in Compliant Motion,* Proceedings of ASME Computer in Engineering Conference, 1986, Chicago, IL

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S. Payandeh, A. A. Goldenberg, *A Robust Grasping Wrench Controller,* (in preparation)

Internal Reports

- Application of the Black-Board Architecture to System Control, Robotics and Automation Laboratory, University of Toronto
- Design of the RAL Direct-Drive Arms, Robotics and Automation Laboratory, University of Toronto
- Application of a Vision System to the Printed Circuit Board Alteration Problem, Robotics and Automation Laboratory, University of Toronto
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- Application of a Vision System to the Identification of Object Location, Center for Flexible Manufacturing, McMaster University

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ABSTRACT

The objectives of this research are: i) to model the contact between the fingers of a dexterous mechanical hand with soft finger-tips and an object; ii) to devise a control law such that the actual linear impedance model of each grasping finger is *matched* with a corresponding desired impedance; iii) to develop a control law such that the contact wrench between each grasping finger and the object is maintained during a task performed, in the presence of parameter uncertainty and disturbances of a certain class.

Using the *causality* principle, each grasping finger is modelled as a system consisting of a mass, a spring and a damper for each direction of the end-point reference coordinate frame. In this model, the spring and damper are connected in parallel between two masses modelling the finger and the palm of the hand respectively. Similarly, the soft finger-tip is modelled as a spring and damper connected in parallel between masses modelling the finger and the grasped object respectively. The desired model of the finger including its mass, spring and damper parameters is referred to as the *targeted impedance.*

In general, the actual linear model of each grasping finger is different from the desired model specified by the targeted impedance. In order to achieve the desired impedance model the concept of impedance matching and a method for its implementation are proposed. The implementation of the impedance matching concept is based on the linear dynamics decoupling approach. It is shown that when the model parameters of the finger are precisely known, the implementation of this method generates the replacement of the actual impedance model of the finger with the targeted impedance.

When fingers of a dexterous mechanical hand are grasping an object, they must exert

wrenches equal to certain desired grasping wrenches. Furthermore these wrenches must be maintained during the task and in the presence of uncertainties and disturbances. Using the matched impedance model of the finger and the model of the soft finger-tip, a grasping wrench control architecture is developed. It is shown that in the presence of uncertainties in the actual parameters of the finger, and of disturbance wrenches, the grasping wrench controller could not have the desired performance. Based on the *theory of the servomechanism problem,* the proposed control architecture was modified such that the system exhibits robustness properties. The modified architecture generates the property that the error between the actual and desired grasping wrenches approaches zero asymptotically even when there are uncertainties in the parameters of the actual finger, and disturbances of a certain class(constant or sinusoidal). The performance of the robust controller is demonstrated through simulation, and shown experimentally using a 2*DOF* planar finger with soft finger-tip making contact with a rigid wall. The performance is also illustrated using a robust grasping wrench controller for tooling tasks. The robustness is obtained by including a model of the *exogenous* disturbance wrench, which arises from the interaction of the tool with the environment, in the control architecture.

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Last but not the least, I would like to thank Miss Eniko Rozsa for helping me with the figures and her expert advice on the software development related to my experiments.

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LIST OF NOMENCLATURE

A, B, C, D = system matrices.

$\mathbf{C}_{()}$ = diagonal damping matrix.

$\mathbf{C}_{fe}$ = actual damping matrix of a finger expressed in the finger end-point reference coordinate frame.

$\mathbf{C}_o$ = *targeted damping* expressed in the object reference coordinate frame.

$\mathbf{C}_t$ = targeted damping of the finger.

$\mathbf{C}_{tip}$ = *targeted damping* expressed in the finger-tips coordinate frames.

$\mathbf{C}_u$ = damping property of a *soft* finger-tip.

G = Grasp Matrix.

$\mathbf{H}_t$ = contact configuration model based on the *twist* representation.

$\mathbf{H}_t^c$ = concatenated contact configurations model

$\mathbf{H}_w$ = contact configuration model based on the *wrench* representation.

$\mathbf{H}_w^c$ = concatenated contact configurations model

I = identity matrix.

J = Grasp Jacobian Matrix.

$\mathbf{J}_\theta$ = a finger Jacobian.

$\mathbf{J}_\theta^c$ = concatenated Jacobian of fingers.

$\mathbf{K}_{()}$ = diagonal stiffness matrix.

$\mathbf{K}_{fe}$ = actual stiffness matrix of a finger expressed in the finger end-point reference coordinate frame.

$\mathbf{K}_o$ = stiffness expressed in the object coordinate frame.

$\mathbf{K}_{ser}$ = gain matrix on the state of the *servo-compensator*.

$\mathbf{K}_t$ = targeted stiffness of a finger.

$\mathbf{K}_{sta}$ = gain matrix on the state of the system.

$\mathbf{K}_{tip}$ = stiffness expressed in the finger-tips coordinate frames.

$\mathbf{K}_u$ = stiffness matrix of the *soft* finger-tip.

$\mathbf{M}_{()}$ = diagonal mass matrix.

$\mathbf{M}_{fe}$ = actual inertia matrix of a finger expressed in the finger end-point reference coordinate frame.

$\tilde{\mathbf{M}}_{fe}$ = approximate model of inertia matrix of a finger expressed in the finger end-point reference coordinate frame.

$\mathbf{M}_t$ = desired mass model of a finger.

$\mathbf{Z}_d$ = decoupled impedance of a finger with *soft* finger-tip.

$\mathbf{Z}_f$ = actual impedance of a finger.

$\mathbf{Z}_s$ = impedance representation of the *matching* condition.

$\mathbf{Z}_s^*$ = known resultant impedance of a finer.

$\mathbf{Z}_t$ = *targeted impedance* of a finger.

$\mathbf{Z}_u$ = impedance of the *soft* finger-tip.

L, M, N, P, Q, R = *Plüker* line coordinates.

S = a *screw*.

T = a *twist*.

T_c = a *twist* expressed in contact reference coordinate frame.

$\bar{T}_c$ = combined vector of twists expressed in the contact reference coordinate frames.

T_{ep} = end-point twist.

T_{fe} = twist of a finger end-point.

T_{fe}^{*} = the desired twist of a finger end-point.

T_o = a *twist* expressed in object reference coordinate frame.

T_{oc} = a twist of the object at the contact area.

$\overline{T}_{oc}$ = combined vector of twists of the object at contact areas.

T_{tip} = a *twist* expressed in finger-tip reference coordinate frame.

T_θ = vector of joint velocities.

W = a *wrench.*

W_d = disturbance wrench.

W_{ext} = external wrench.

W_{act} = actuating wrench expressed in the end-point reference coordinate frame.

W_{act}^{t} = actuating wrench as function of *targeted impedance.*

W_g = grasping wrench.

W_g^{*} = desired grasping wrench.

W_c = a *wrench* expressed in contact reference coordinate frame.

$\overline{W}_c$ = combined vector of wrenches expressed in the contact reference coordinate frames.

W_o = a *wrench* expressed in object reference coordinate frame.

W_{oc} = a *wrench* expressed in object/contact reference coordinate frame.

$\overline{W}_{oc}$ = combined vector of wrenches expressed in the object-contact reference frames.

W_{tip} = a *wrench* expressed in finger-tip coordinate frame.

$\overline{W}_{tip}$ = combined vector of the wrenches expressed in the finger-tips coordinate frames.

W_θ = vector of joint torques.

e = error vector.

f = force vector.

m = dimension of input control vector u.

m_o = moment vector.

mt = dimension of space of *twist-of-constraint.*

mw = dimension of wrench-space of contact area .

n = dimension of the state vector x.

nt = dimension of space of twist of a finger-tip or object at contact area.

ntt = total dimension of the spaces of twists of finger-tips or object at contact areas.

nw = dimension of wrench-space of finger-tip or object at contact area.

nwt = total dimension of the spaces of wrenches of the finger-tips or object at contact areas.

r = dimension of the output vector y.

s = *Laplace* operator.

$s_1, s_2, s_3, s_4, s_5, s_6$ = components of a *screw.*

t = time.

$t_1, t_2, t_3, t_4, t_5, t_6$ = components of *twist.*

u = input control vector.

v = velocity vector.

$w_1, w_2, w_3, w_4, w_5, w_6$ = components of a *wrench.*

x = state vector of a system.

y = output vector of a system.

y_{ref} = reference output vector.

β^* = system matrix of a *servo-compensator*.

Λ = system matrix of a *servo-compensator*.

Ψ = companion matrix of the *exogenous* inputs.

α_p = coefficient of polynomial.

ε = dual operator.

η_t = *intensity* of a *twist*.

η_w = *intensity* of a *wrench*.

μ = eigenvalue.

ν = transmission zero of a system.

ρ_s = *pitch* of a *screw*.

ρ_t = *pitch* of a *twist*.

ρ_w = *pitch* of a *wrench*.

η_t = *intensity* of a *twist*.

η_w = *intensity* of a *wrench*.

ξ = state vector of a *servo-compensator*.

ω = disturbance.

ω_d = angular frequency of disturbance .

ω_t = angular frequency of excitation force.

ω_w = angular frequency of the dynamic intensity.

$\frac{d}{dt}(.)$ = differentiation with respect to time t.

$\int(.)dt$ = integration over a period of time t.

$\in$ = member of.

$\|(.)\|$ = norm of (.).

$\frac{\partial(.)}{\partial(.)}$ = partial derivative.

$(.)^T$ = transpose of (.).

$\delta(.)$ = variation of (.).

Myself, when young did eagerly frequent ;

Doctor and Saint, and heard great Arguments ;

About it about : but evermore ;

Came out the same Door, as in I went .

Omar Khayyam

CHAPTER I
Introduction

This chapter presents the motivation, objectives and contributions of the thesis. It is organized as follows: section 1.1 describes the motivation of the thesis; section 1.2 outlines the main areas of research in *dexterous mechanical hands;* section 1.3 presents the review of the related literature and section 1.4 states the contributions of the thesis. Finally, section 1.5 presents the organization of the remaining chapters in this thesis.

1.1 Motivation of the Thesis

Advanced applications of *a manipulating system* for automating the manufacturing *work-cells* require an in-depth understanding and fundamental research into the various issues arising from the current applications to manufacturing work-cells.

A typical automated manufacturing work-cell usually has a manipulator which can perform a number of tasks, Goldenberg and Payandeh[1], Figure 1.1. These tasks can be divided into two basic categories:

a) *pick-and-place* tasks,

b) *tooling* tasks.

The first category consists of tasks where a manipulator has to pick an object from a *pick* location and put it at a given *place* location. These tasks, may involve various types of

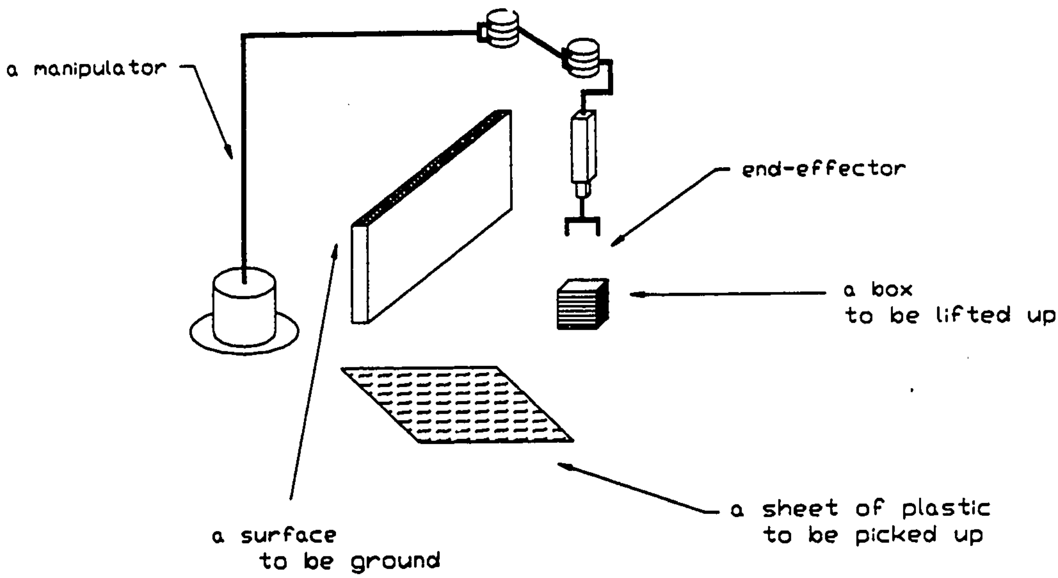


Figure 1.1 – A manipulator with various tasks assigned to its work-cell.

objects with different geometric attributes. Hence, the *end-effector* of the manipulator must be changed frequently in order to grasp different shaped objects at the pick locations. As a result, in the sequence of the operation, delays are introduced to allow enough time to interchange the end-effectors and this increases the total operation time. The change of end-effectors becomes a necessity when the manipulator has to perform a tooling task, e.g. surface finishing task, in addition to the pick-and-place task and hence the presence of the delays in the sequences of operations becomes inevitable.

One possible solution to the above problem is to attach a *multi-end-effector* to the end-plate of the manipulator. In principle, a multi-end-effector consists of number of specialized end-effectors and specialized *tools* for performing different tooling tasks, all combined as a single unit. For example, to pick-up an object, a designated *end-effector* is selected and located on top of an object at a pick location and then, for performing a tooling task, a designated tool is selected. Figure 1.2 shows an example of a *dual end-effector* where a gripper and an automatic screw driver are attached through a common based to the end-plate of the manipulator. Pick-and-place task can be performed by selecting the gripper and noting its relative location with respect to the end-plate reference coordinate frame and *screw driving* task can be performed by selecting the screw driver.

A multi-end-effector offers a solution to each specialized work-cell of a factory, however, this implies that a number of dedicated multi-end-effectors should be available. As a result, if the overall production objectives of the automated work-cells are modified, the dedicated multi-end-effectors will have to be discarded.

A natural extension from a multi-end-effector to a more advanced concept which does not have the above mentioned disadvantages is a *dexterous mechanical hand.* The main advantage of a dexterous mechanical hand is its universality. This means that the *hand* can be used to pick up various types of objects with different attributes regardless of the category of task, i.e. pick-and-place or tooling tasks.

Another application of a dexterous mechanical hand can be found in the

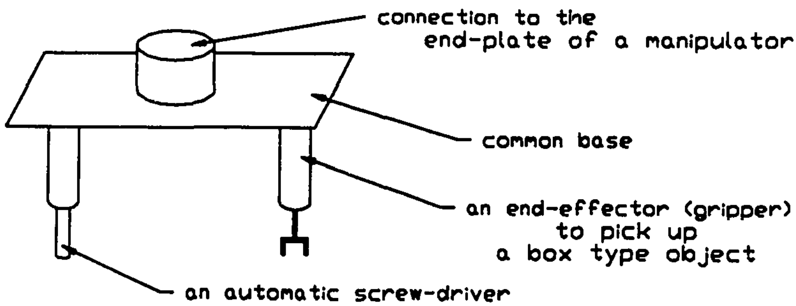


Figure 1.2 – A dual end–effector, Goldenberg and Payandeh[1].

biomechanical area where the hand can be used as a replacement of a person's natural hand if a part of or all of his/her hand had been lost. Current replacements consist of an end-effector which resembles two fingers of a human hand with very limited dexterity. These replacements although practical, do not satisfy the original needs for dexterity of a disabled person for manipulating his/her environment.

1.2 Dexterous Mechanical Hands

A *dexterous mechanical hand* is an end-effector whose mechanical structure and modes of operation resemble those of a human hand. In principle, this mechanism consists of a base, i.e. a *palm,* where a number of open kinematic chains, i.e. *fingers,* are connected to, see Figure 1.3. Each finger may consist of a number of joints, i.e. revolute and/or prismatic, which are actuated either directly or indirectly. Direct actuation of each joint of a finger is referred to as a method where the actuators are located either in each joint or in the palm and they are connected to a finger through a closed-kinematic chain, Hunt and Torfason[2], Asada and Youcef-Toumi[3], Figure 1.4. Indirect actuation is referred to a configuration of a hand where each joint of a finger is actuated through drive mechanisms which are connected to the actuators located external to the hand, Salisbury[4]; Jacobsen, Wood, Knutti and Biggers[5]; Goldenberg and Kim[6], Figure 1.5.

Application of a dexterous mechanical hand to automated manufacturing work-cells is still in its infancy. This is due to the following issues which could be considered as research objectives in various engineering, applied mathematics and computer science fields , Payandeh and Goldenberg[7]:

- Mechanical Configuration
- High-level Control
- Low-level Control

In *mechanical configuration,* compactness and lightness of the design are the main

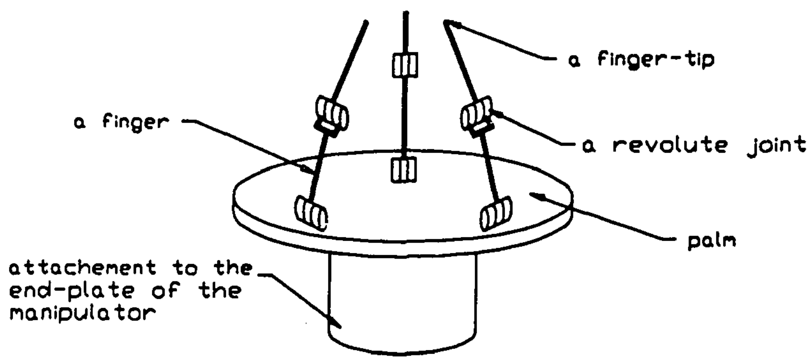


Figure 1.3 – A schematic of a dexterous mechanical hand.

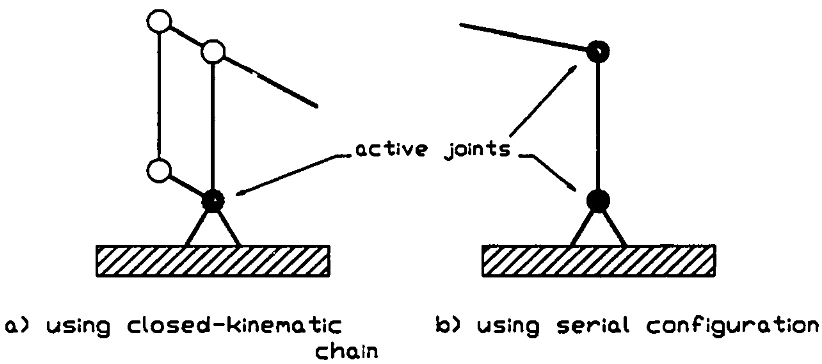


Figure 1.4 - Schematic of directly actuated finger's joints.

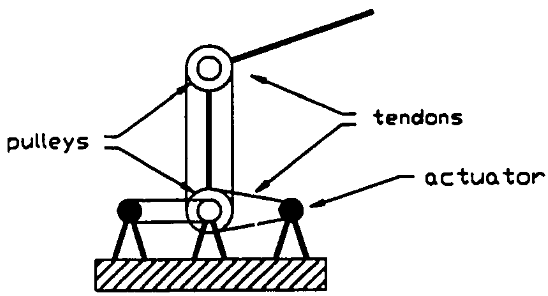


Figure 1.5 – Indirect actuation of the joints of a finger using tendon mechanisms.

issues. Designing a hand where the complete actuating system can be fitted in the palm of the hand, or in the joints of a finger, and still being light, is the major concern in this area.

High-level control is a decision making process for performing tasks which are assigned to a work-cell of a dexterous mechanical hand. These hierarchical decision making stages are defined as follows:

a) arm/hand system *path planning,*

b) hand *pregrasp* configuration,

c) *grasping,* and

d) *manipulation*

Definition 1.1: *Path-planning* is a decision making process for determining a collision free path of the arm/hand system in order to accomplish a given task, Figure 1.6. The planning utilizes various external knowledge sources which may be supplied by various external sensing systems, e.g. a vision system.

Definition 1.2 : *Pregrasp configuration* of a hand is a posture which depends on the attributes of an object and types of task assigned to a work-cell and it is suitable for accomplishing the given task. This configuration can determine the number of fingers to be used and the pose that each finger must have when approaching the object to be grasped, Payandeh and Goldenberg[8].

Definition 1.3 : After approaching and pregrasping, *grasping* is the process of making and maintaining contact(s) with an object without change of contact points during a preassigned task.

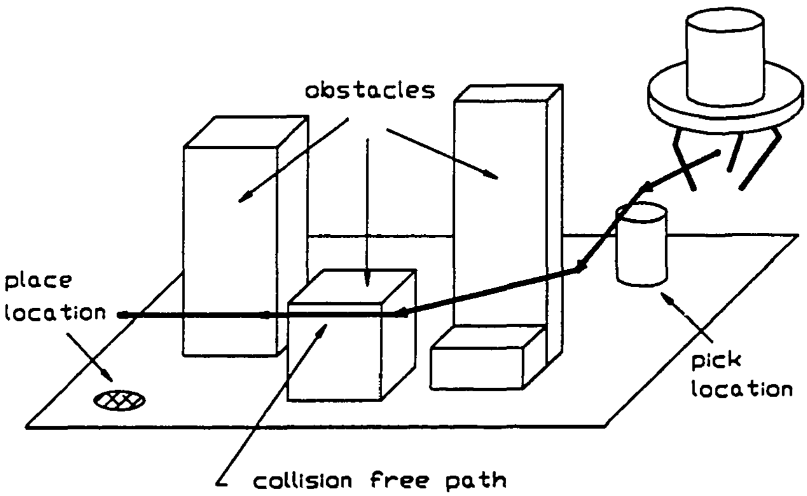


Figure 1.6 – A collision free path of the arm/hand system.

Definition 1.4 : *Manipulation* is the movement of grasping fingers which can move the grasped object with respect to the *palm.* Manipulation of the grasped object is further divided into two categories: a) *fine manipulation* and b) *coarse manipulation.*

Definition 1.4.1 : *Fine manipulation* is referred to as the relocation of the grasped object with respect to the palm where fingers of a dexterous mechanical hand maintain contact with the object. Rolling an object between the fingers is an example of *fine manipulation,* Hui and Goldenberg[9], Figure 1.7.

Definition 1.4.2 : *Coarse manipulation* is referred to as the relocation of the grasped object with respect the palm where the fingers of a dexterous mechanical hand periodically lose and gain contacts with the grasped object . For example, relocation of the pencil between fingers when it is grasped from the wrong end, Figure 1.8.

The decisions of the high-level control, e.g. finger-tip grasping forces and moments, are passed to the *low-level control* for execution. The objectives of the *low-level control* are to execute and maintain these command vectors in the presence of uncertainties and unwanted input disturbances acting on the dexterous mechanical hand.

1.3 Related Literature

The thesis is concerned with the area of *grasping*. Specifically, the objectives of this research are: i) to model the grasp between the fingers of a dexterous mechanical hand with soft finger-tips and an object; ii) to devise a linear control law such that the actual linear impedance of each grasping finger is *matched* with the desired impedance; iii) to develop a control law such that the contact between each grasping finger and the object is maintained during a task and the presence of uncertainties.

The research in the area of *grasping* has been divided into three basic subareas:

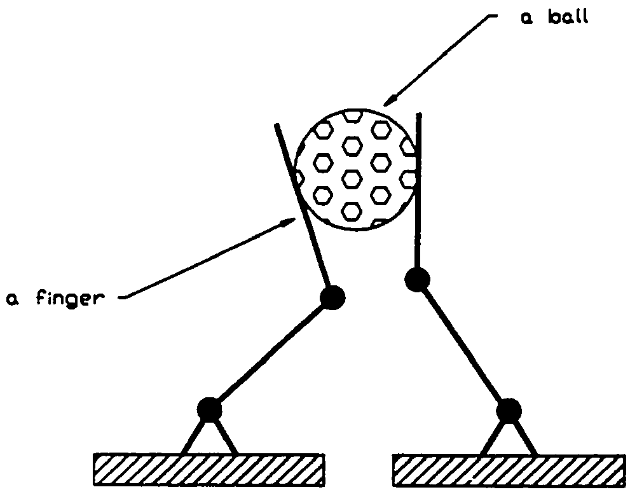


Figure 1.7 – Example of a fine manipulation.

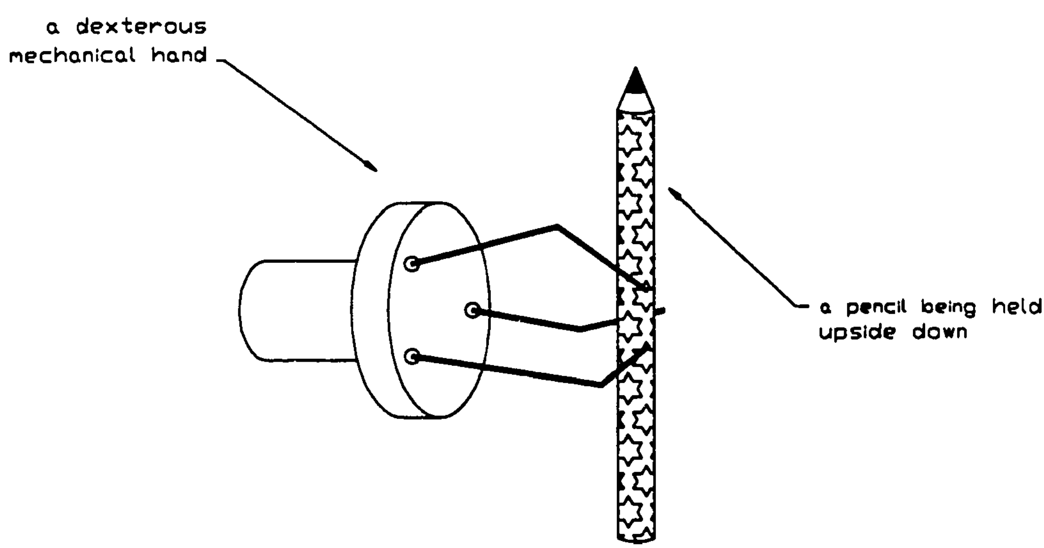


Figure 1.8 – Example of a coarse manipulation.

a) methods for determining the directions and magnitudes of grasping forces between the finger-tips and the grasped object,

b) methods for determining the response of the *grasp* to the external forces acting on the grasped object, i.e. determining grasp properties, and

c) methods for controlling the grasp such that the object does not slip between fingers during a task execution

The following is a review of some of the major research in these subareas:

Research in the area of determining the magnitudes and the directions of grasping forces have mostly been based on the assumption that there are three grasping fingers which make point contact with friction with the object. Using screw theory and geometric representation of the object, Holzmann and McCarthy[10] proposed a method which ensures condition that prevents the grasped object from slipping under its own weight between the fingers. This method can also be extended to determine the optimal contact configuration that fingers can make with an object. Yoshikawa and Nagai[11] proposed a method which does not require the knowledge of the external forces acting on the grasped object, i.e. weight of the object. The method used a special property of the grasp matrix which permits the determination of the maximum magnitude of the grasping force that each finger can exert on the grasped object. Ji and Roth[12] proposed a method for computing grasping forces based on optimization technique and geometric reasoning. In this method the objective is to minimize the maximum angle between the direction of grasping force and the surface normal at the contact areas. Podhorodeski, Fenton and Goldenberg[13] proposed a method which analytically determines the bases of the null space of the grasp configuration matrix and extended the method of Ji and Roth[12] for cases where the out-of-plane surface normals are also considered for determining the optimal grasping forces. Demmel and Lafferriere[14] generalized the method of Ji and Roth[12] and reduced the nonlinear optimization method to the generalized eigenvalue problem which can be solved easily. The method was also extended to spatial grasps.

In the subarea of determining properties of grasp, based on the method for controlling a manipulator in contact with an environment, e.g. stiffness control, Mason and Salisbury[18] suggested that the grasped object should have the property of spring. For point contact with friction and a model of the grasp matrix, the desired spring property of each finger is represented in the finger reference coordinate frame. Nguyen[15] proposed that for a given contact configuration where the fingers make point contact without friction, the grasp should have spring property so when it is displaced from an equilibrium configuration, it returns to this configuration. Li and Sastry[16] defined a number of grasp quality measurements for optimization which will determine an optimal grasp configuration of each finger given a model of tasks assigned to the grasped object. Cutkosky and Kao[17] discussed sources of the spring property of the grasp.

The overall control objective of the grasp controller is to control the position of the fingers and their contacting forces relative to the grasped object. Mason and Salisbury[18] proposed a method for controlling the position of each finger. The method is based on the stiffness control approach which can control the grasping forces between fingers and the grasped object implicitly. Arimoto, Miyazaki and Kawamura[19] proposed a master/slave control approach for position control of each finger. In this approach fingers are grouped such that a group of slave fingers can follow the controlled motion of a group of master fingers. The object is held between grasping fingers by ensuring that relative distances between fingers are maintained, i.e. implicit method of grasping force control. Furthermore, stability of the controller is ensured using the Lyapunov method. Li and Sastry[16] proposed a method where each finger is controlled depending on the other fingers. In this method both positions and grasping forces between fingers and the object are controlled, i.e. the grasping forces are controlled explicitly. The grasping forces are assumed to be measured using a force measuring transducer.

1.4 Contributions of the Thesis

In general, this thesis is concerned with the modelling of the interaction between the fingers and the grasped object, and the design of a linear control law such that the contact between a finger and the object is maintained during a task. The major contributions of the thesis are highlighted as follows:

i) An approach to model the contact between the grasping fingers with soft finger-tips and the object is proposed *(section 2.1)*. The approach is based on the *causal* representation of mechanical elements. This approach leads to simple models.

ii) Concept of *impedance matching* which is different than the concept of impedance control, Hogan [20], and a method for its implementation is proposed(*chapter VI)*. In impedance matching the objective is to design a control law such that the actual impedance model of a finger is replaced by, i.e. matched with, a desired impedance, whereas in impedance control the objective is to design a control law such that a desired impedance model of the contacted environment is implemented.

iii) Using the concept of *impedance matching* and the model of the soft finger-tip, an architecture for independent control of the grasping forces and moments, i.e. *grasping wrenches,* of each finger is proposed, see *section 3.3*.

iv) In order to ensure that the grasping wrench controller has the desired performance based on the *theory of the servomechanism problem,* a robust control architecture is proposed. With this controller the response is asymptotically stable when constant or sinusoidal disturbance wrenches are acting on the system and when variations in the dynamic parameters of the finger occur(*section 3.6)*. The theory is illustrated using an example of a tooling task performed with a grasped tool(*section 3.7)*.

v) The performance of the independent control of the contact wrench of a finger is demonstrated through experiments using a *2DOF* direct-drive planar finger with soft finger-tip making contact with a rigid wall. The experimental results agree with the simulated results(*section 4.3)*.

1.5 Thesis Organization

The thesis is organized as follows: chapter *II* postulates a desired model of a grasping finger referred to as the *targeted impedance* and proposes a method for *matching* the actual linear impedance model of a finger with the targeted impedance; chapter *III* proposes a robust controller, using the matched impedance model of a finger, for controlling the grasping wrench between the finger and the object; chapter *IV* presents the experimental verification of the robust controller, using a *2DOF* finger, making soft contact with a rigid wall, and chapter *V* concludes the thesis and outlines future research.

CHAPTER II
Impedance Matching

In the previous chapter, *definition 1.3* stated grasping as the process of making and maintaining contact with the object using mechanical fingers. object. In this chapter the model of a grasping finger is postulated. The desired model of the grasping finger is referred to as the *targeted impedance*.

In order to grasp an object, each finger moves infinitesimally from its initial contact point with the object. The dynamic model of the finger during the grasping process can be assumed to be linear about the initial contact point of the finger with the object.

In general, the actual linear impedance model of each grasping finger is different from the desired model specified by the targeted impedance. In order for the actual grasping finger to have the desired impedance, the concept of *impedance matching* is proposed with the method of its implementation.

For the purpose of clarity, the model of each grasping finger is presented as a one-dimensional model. However, the model is easily extendable to multi-dimensional system and to many grasping fingers since the approach used in control is decoupled dynamics. The chapter is organized as follows: section 2.1 presents a model of grasping fingers which form a grasp; section 2.2 presents the concept of impedance matching; section 2.3 proposes a method for implementing the concept while section 2.4 presents a summary on the results of this chapter.

2.1 Grasp Model

This section presents a one dimensional model of a grasping finger. The results of this section are easily extendable to a multi-dimensional model of all grasping fingers which form the grasp.

Assumption 2.1: The palm of the hand and the objects to be grasped are rigid bodies. The finger-tip is compliant, i.e. soft, having spring and damping properties.

Definition 2.1: The port of interaction, see *definition A.6,* between two elements of the mechanical system is a causal one when the input/output of the elements at this port are matched, i.e. the output of one element is the input to the other or vice versa.

The following proposition states that the port of interaction between a spring and a mass element is causal.

Proposition 2.1: The port of interaction between a spring and a mass element of a mechanical system is a *causal* port of interaction.

Proof: The *causal* representation of a spring element has an *across* variable, e.g. a twist, as an input and a *through* variable, e.g. a wrench, as an output, see *section B.1*. Similarly, the *causal* representation of a mass element is defined as having a *through* variable as an input and an *across* variable as an output. Therefore, when a mass and a spring are in contact, their corresponding inputs/outputs at their port of interaction are matched. □

Corollary 2.1: The port of interaction between two mass elements is not a causal one. Referring to the causal representation of the mass element, figure B.2, when two mass elements are brought into contact, both elements require force as an input and produce velocity as an output. As a result, since the output of one element is not the input to the other or vice versa, the port of interaction between two mass elements is not a causal one.

Corollary 2.2: The port of interaction between a damper element and a mass element is a causal one.

Corollary 2.3: The port of interaction between the grasped object and the environment is not a causal one since both bodies are assumed to be rigid, i.e. two mass elements.

The following *propositions* first define the model of each grasping finger excluding the model of soft finger-tip and then the model of each grasping finger with soft finger-tip, see *assumption A.2*.

Proposition 2.2: The entire model of each grasping finger, excluding the soft finger-tip, must be modelled as a mass, spring and damper model where the spring and damper are connected in parallel between the mass model of the finger and the palm of the hand.

Proof: The proof is based on showing that any other combination of the mass, spring and damper models of a finger is not a *causal* combination but the combination in the statements of the proposition. Figure 2.1a shows a schematic of only a mass model of a finger in contact with the palm of the hand. From *corollary 2.1* it follows that if the finger is modelled as a mass in contact with the palm, i.e. another mass, their port of interaction is not a *causal* one, therefore, modelling the grasping finger only as a mass element is not a causal model.

Next let us now consider the model of a finger to be a spring in series with a damper connected between the mass model of the finger and the palm of the hand, see Figure 2.1b. From *proposition 2.1* it can be stated that in general, the port of interaction between the spring and mass is a causal port. Also, the port of interaction between the damper model and a mass model is causal, see *corollary 2.2*. However, the port of interaction between

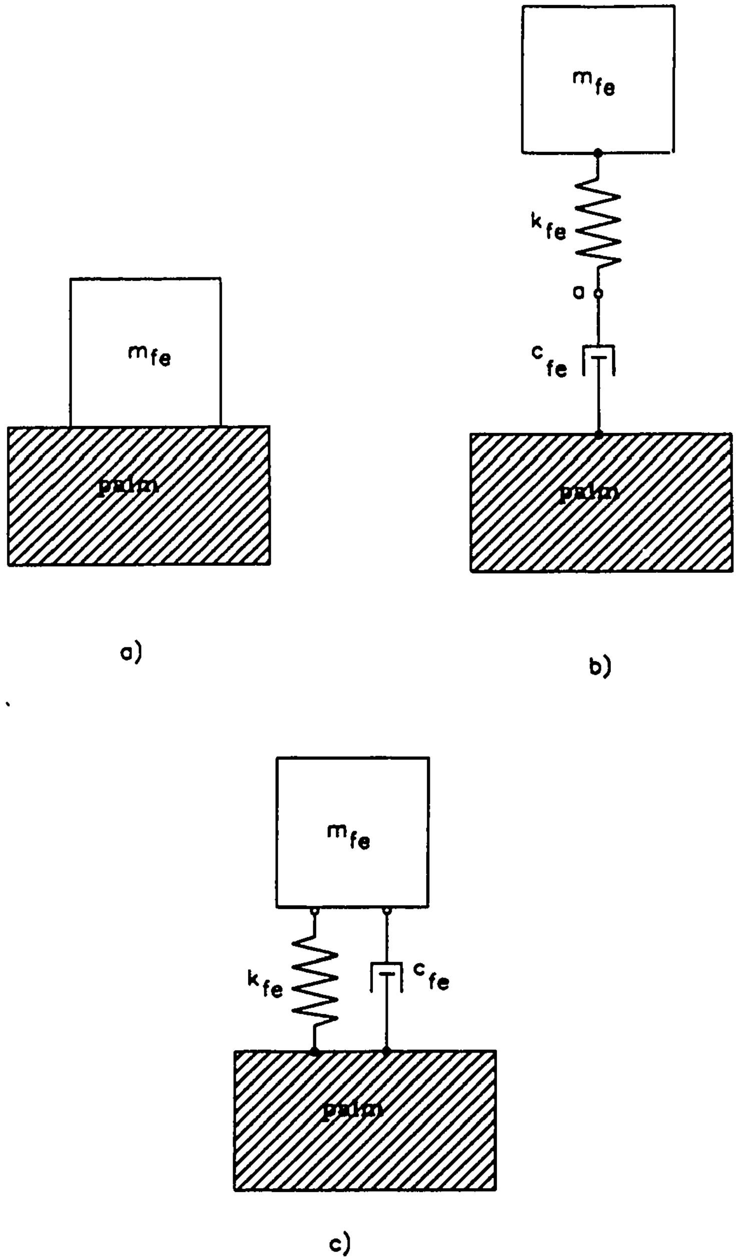


Figure 2.1 – Models of a finger which exclude the model of the soft finger-tip.

the spring and the damper elements is not causal, i.e. port *a* in the Figure 2.1b. At this port, the spring element requires twist as an input. The damper element is represented as having wrench as an output because of its interaction with the palm. Therefore, at port *a*, the damper element has wrench as an output variable which does not match with the input requirements of the spring element. As a result, the model of Figure 2.1b is not a causal model.

Figure 2.1c shows the causal model of the finger. In this figure the ports of interaction between the spring and the masses of the finger model and the palm are causal ones. Also, the ports of interaction of the damper element connected in parallel between the spring element with the two masses are causal. As a result, the finger model of Figure 2.1c is the only causal model. □

The following *proposition* gives the model of each grasping finger which includes the soft finger-tip in contact with the grasped object.

Proposition 2.3: Given the model of a finger which excludes the soft finger-tip, *proposition 2.2*, the model of the soft finger-tip is given as the spring and damper models of the finger-tip connected in parallel between the grasped object and the mass model of the finger.

Proof: The proof is similar to the proof of *proposition 2.2*. Let Figure 2.2a represents a schematic model of the soft finger-tip in contact with the finger mass model and the grasped object. In this figure, the port of interaction between the spring model of the soft finger-tip and the grasped object is a causal one. Also, the port of interaction between the damper model of the soft finger-tip and the mass model of the finger is causal. However, since at the port of interaction *a*, the input/output of the spring and the damper does not match, this model is not a causal model.

Figure 2.2b shows the causal model of the finger and the finger-tip in contact with the grasped object. As it can be seen in this Figure, the spring and the damper element of the

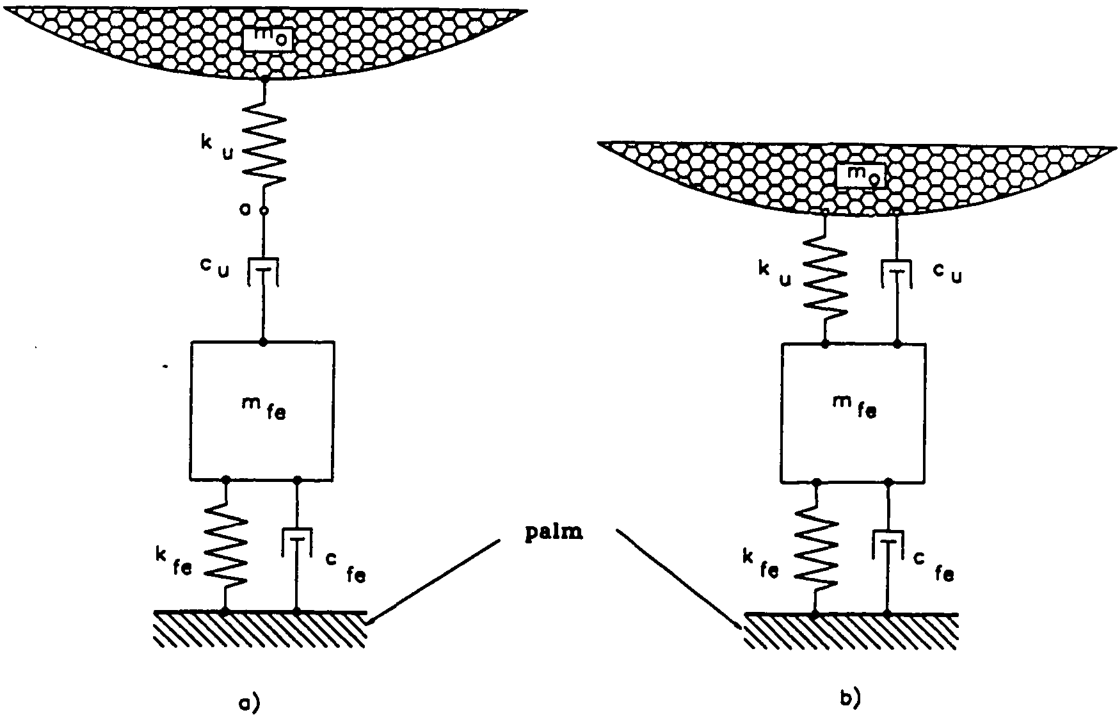


Figure 2.2 – Models of a finger with soft finger–tip in contact with the grasped object.

soft finger-tip are connected in parallel between the grasped object and the mass model of the finger. □

The above propositions postulated a model of the grasping finger based on causal representation of mechanical elements. This generated a simple and intuitive approach to modelling the finger and the finger-tip in contact with the grasped object. Different models can be obtained based on non-causal representation of mechanical elements. However, the discussion on the implications of these non-causal models is beyond the scope of this thesis.

The parameters of the model of each grasping finger can be selected such that the responses of all the grasping fingers can contribute in the desired response of the object to the external forces and moments. For example, for contact tasks, it is shown in *section c.1* that the grasped object must have spring or damper models at the port of interaction with the environment. Relationships between the spring and damper models of the fingers expressed in the finger-tips reference coordinate frames and the spring and damper models of the object expressed in the contact reference coordinate frame are developed. In general, the parameters of the spring and damper models of each finger can be selected such that the grasped object has the desired parameters at the port of interaction with the environment. Another example is when holding a vibratory object, *section c.2*. In this case, a method for determining the relationships between the model parameters of the grasping fingers is developed such that the vibration of the grasped object can be reduced to zero.

Throughout the remainder of this thesis it is assumed that these parameters are given and the model is referred to as the *targeted impedance* of a finger $\mathbf{Z}_t$. In general, the impedance model represents a relationship between the force and displacement, Hogan[20].

2.2 Impedance Matching Concept

This section presents the concept of *impedance matching* which establishes an objective foı designing linear feedback control laws to control a finger of a dexterous mechanical hand with soft finger-tip in contact with the object.

The notion of *impedance matching* is used in various engineering fields. For example, in electric circuit theory, the objective is to maximize the power flow between the main power line and the load by designing an intermediate circuitry, e.g. either in parallel or in series, which matches the load impedance to the main line impedance, Seshadri[21]. Also, this notion is used in the vibration control of a beam, Van de Vegte[22].

In general, the actual linear impedance model of a finger given by $\mathbf{Z}_f$ is different from the impedance model specified by the targeted impedance $\mathbf{Z}_t$. *Impedance matching* is referred to as the objective of replacing $\mathbf{Z}_f$ with $\mathbf{Z}_t$. In this thesis this replacement(or matching) is achieved using linear feedback control.

The concept of impedance matching is different from the concept of impedance control which was proposed by Hogan[20]. Unlike the impedance matching concept, proposed in this thesis, in impedance control the objective is to design a feedback control law such that a desired impedance model of the contacted environment is obtained.

2.3 Implementation of the Impedance Matching Concept

This section presents the method for implementing the concept of impedance matching.

Assumption 2.2: The actual dynamic parameters of a finger with the soft finger-tip are known and there is no disturbance wrench acting on the finger.

Assumption 2.3: During grasping, the palm of the hand is fixed with respect to the object reference coordinate frame.

Assumption 2.3: During grasping, the palm of the hand is fixed with respect to the object reference coordinate frame.

The actual linear dynamic model of a finger in contact with the grasped object expressed in the finger end-point reference coordinate frame is written as, see Appendix D :

$$\mathbf{M}_{fe}\dot{T}_{fe} + \mathbf{C}_{fe}T_{fe} + \mathbf{K}_{fe}\int T_{fe}dt + W_{ext} = W_{act} \ , \qquad (2.1)$$

where based on *assumption 2.2,* the actual parameters of the finger namely, $\mathbf{M}_{fe}, \mathbf{C}_{fe}$ and $\mathbf{K}_{fe}$ are assumed to be known. W_{ext} is the external wrench acting on the finger, W_{act} is the actuating wrench defined in the finger end-point reference coordinate frame and T_{fe} is the twist of the finger end-point, see section A.2.

In mechanical model representation, a wrench is defined as a through variable in the elements of the system. In equation 2.1, W_{ext} is the external wrench acting on the finger which is also the wrench defined in the contact reference coordinate frame. The bases of this wrench are defined by the constraining-wrenches, see *definition A.11,* which are the bases of the grasping wrench W_g. Therefore, we can write:

$$W_{ext} = W_g \ .$$

The grasping wrench W_g is calculated from the knowledge of the model of soft finger-tip, see *proposition 2.3,* as:

$$W_g = \mathbf{C}_u T_{fe} + \mathbf{K}_u \int T_{fe}dt \ . \qquad (2.2)$$

Substituting the above equation into equation 2.1 we have:

$$\mathbf{M}_{fe}\dot{T}_{fe} + (\mathbf{C}_{fe} + \mathbf{C}_u)T_{fe} + (\mathbf{K}_{fe} + \mathbf{K}_u)\int T_{fe}dt = W_{act} \ , \qquad (2.3)$$

or, the actual impedance of the finger in parallel with the impedance of the soft finger-tip where $\mathbf{Z}_f = \mathbf{M}_{fe}s^2+\mathbf{C}_{fe}s+\mathbf{K}_{fe}$ and $\mathbf{Z}_u = \mathbf{C}_u s+\mathbf{K}_u$.

The method of implementation of impedance matching is based on the method of linear dynamic decoupling control law.

Definition 2.2: A *linear dynamics decoupling* control law or computed torque method, Goldenberg[23], consists of two parts. The first part is the actual linear model of the finger which when implemented cancels the coupled dynamic parameters of the finger and the second part is a decoupled linear control law.

Based on the above definition, the control law for implementing the impedance matching concept is written as:

$$W_{act} = \left[\mathbf{M}_{fe}W^e_{act}+\mathbf{C}_{fe}T_{fe}+\mathbf{K}_{fe}\int T_{fe}dt+W_g\right] , \tag{2.5}$$

where W^e_{act} is the linear control law for the decoupled model of the finger with soft finger-tip in contact with the object. Substituting the above control law into equation 2.3 we have:

$$\mathbf{M}_{fe}\dot{T}_{fe}+\mathbf{C}_{fe}T_{fe}+\mathbf{K}_{fe}\int T_{fe}dt+W_g = \mathbf{M}_{fe}W^e_{act} + \mathbf{C}_{fe}T_{fe}+\mathbf{K}_{fe}\int T_{fe}dt+W_g . \tag{2.6}$$

Simplifying the equation, we obtain the following dynamic model, dynamics of the inner-loop, Figure 2.3,

$$W^e_{act} = \dot{T}_{fe} = \frac{1}{s}\mathbf{Z}_d T_{fe} , \tag{2.7}$$

where

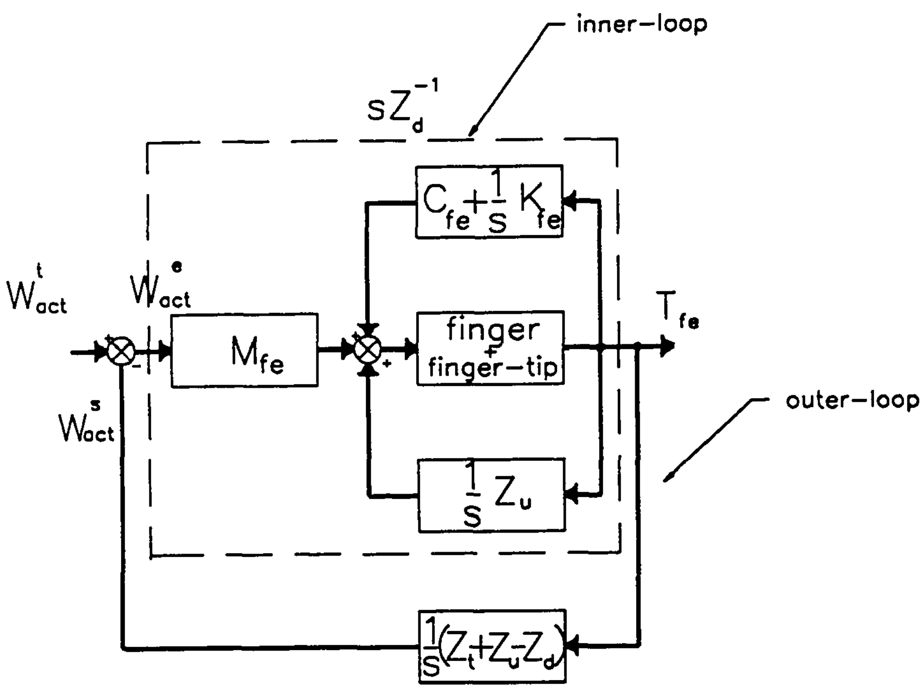


Figure 2.3 – Control block diagram of the implementation of impedance matching concept.

$$\mathbf{Z}_d = s^2\mathbf{I} \ .$$

$\mathbf{Z}_d$ is referred to as the feedback decoupled impedance model of a finger with soft finger-tip in contact with the grasped object. In order to cancel the dynamics of the inner loop, the linear controller W^e_{act} is defined as:

$$W^e_{act} = W^t_{act} - \frac{1}{s}(\mathbf{Z}_t + \mathbf{Z}_u - \mathbf{Z}_d)T_{fe} \ , \tag{2.8}$$

where W^t_{act} is the reference actuating wrench and,

$$\mathbf{Z}_s = \mathbf{Z}_t + \mathbf{Z}_u - \mathbf{Z}_d \ , \tag{2.9}$$

the impedance model $\mathbf{Z}_s$ is referred to as a matching condition which includes the targeted impedance, the impedance model of the soft finger-tip and the impedance model of the decoupled model of the finger. Substituting the linear decoupled control law of equation 2.8 into equation 2.7 we have:

$$W^t_{act} = \frac{1}{s}(\mathbf{Z}_t + \mathbf{Z}_u)T_{fe} \ . \tag{2.10}$$

Figure 2.3 shows the control block diagram of the implementation of the impedance matching concept. Comparing equation 2.10 with the actual impedance model of the finger in parallel with the impedance of the soft finger-tip given by:

$$W_{act} = \frac{1}{s}(\mathbf{Z}_f + \mathbf{Z}_u)\, T_{fe} \ , \tag{2.11}$$

it can be seen that the actual linear impedance of the finger $\mathbf{Z}_f$ is replaced with the desired impedance specified by the targeted impedance $\mathbf{Z}_t$.

2.4 Summary

This chapter first postulated a one dimensional linear model for the grasping finger with soft finger-tip. The models are obtained using the principle of causality. It was proposed that the finger must be modelled as a spring and a damper connected in parallel between the mass model of the finger and the palm of the hand. Similarly, the soft finger-tip was modelled as a spring and a damper connected in parallel between the mass model of the finger and the grasped object. However, the one dimensional model easily extendable to multiple dimensions and can also model the interaction of the robotic arm with an environment. The model of a finger where its parameters are given is referred to as the *targeted impedance*.

In general the actual linear impedance model of the finger is different from the desired model specified by the targeted impedance. The second part of the chapter was concerned with the implementation of the impedance matching concept where the objective was to match the actual impedance of a finger with the targeted impedance. A method for implementing this concept was proposed. Based on the assumption that the exact actual model parameters of the finger are available, a control law based on the linear dynamic decoupling was developed. It was shown that implementation of the control law resulted in the actual impedance of the finger to be replaced with the desired impedance specified by the targeted impedance.

CHAPTER III
Robust Control of Grasping Wrench

When a dexterous mechanical hand grasps an object, its fingers must exert the grasping wrenches on the object such that the object is held between the fingers. Furthermore, these grasping wrenches must be maintained during a task and in the presence of uncertainties.

This chapter presents an architecture for controlling the grasping wrench of a finger with soft finger-tip in contact with the object. The control architecture of the grasping wrench takes advantage of the concept of impedance matching which was proposed in the previous chapter.

Based on the *theory of the robust servomechanism problem,* Davison, Goldenberg[24], given a linear model of a system, e.g. finger and finger-tip, in order for a robust controller to exist, the system must satisfy a number of conditions. This chapter examines the existence of the robust grasping wrench controller for a given model of the system and further redefines the feedforward impedance/admittance blocks of the general grasping wrench control architecture such that the architecture exhibits robustness. In addition, based on the general theory, a robust grasping wrench controller is proposed for tooling tasks. For reasons of clarity, the analysis of this chapter is presented using a 2*DOF* finger.

The chapter is organized as follows: section 3.1 outlines some previous published methods to control the contacting wrench between the end-point of a manipulator and the environment; section 3.2 presents the architecture for controlling the grasping wrench;

section 3.3 presents the *nominal* model of a *2DOF* finger with soft finger-tip; section 3.4 discusses the sources of uncertainties in the nominal model; in section 3.5, based on the theory of the servomechanism problem, the existence of a *robust* controller given the nominal model is examined; section 3.6 gives the alternate definitions for the impedance/admittance blocks of the general grasping wrench control architecture which make the controller robust to constant unwanted input disturbance wrenches and variation in the system parameters; section 3.7 extends the general solution to the servomechanism problem for the case when a tooling task is assigned to the grasped object. Finally, section 3.8 summarizes the results of this chapter.

3.1 An Overview of Methods for the Control of Contacting Wrench

There have been numerous published methods for controlling the contact wrench between the end-point of a manipulator and environment. In the area of grasping, these methods can be extended to control the grasping wrench between fingers of a dexterous mechanical hand and the object. However, motivations for selecting these methods are not well defined and selection is heuristic in nature. The following overview outlines some of the published methods for controlling the contacting wrench between the end-point of a manipulator and the environment.

De Schutter and Van Brussel[25] investigated the closed-loop performance of various contacting wrench controllers to step inputs. These are, *integral (I)* or *integral plus derivative (ID)* controllers. Methods for selecting the gain parameters are defined based on the standard methods of *pole-placement* and/or *optimal control,* Davison[26], Van de Vegte[27]. Seraji[28] proposed a method to control the contacting wrench between the end-point of the manipulator and the environment based on *proportional-integral-derivative (PID)* controller, where the gain parameters are selected *adaptively* based on some adaptation law. A control law for controlling the grasping wrenches between the fingers of a dexterous mechanical hand and the object is proposed by Li and Sastry[16].

This controller is a *proportional-integral (PI)* and the gain parameters are selected by using standard methods.

Remark 3.1: Two main points are highlighted from the above :

a) The motivation for selecting a contacting wrench controller is not well defined, i.e. the selection is heuristic in nature.

b) The selection of the gain parameters is based on the standard methods, e.g. pole-placement, optimal control or adaptive algorithms.

The following section presents an architecture, based on the concept of impedance matching for controlling the grasping wrench between the finger-tip and the grasped object. Unlike the other published methods, this architecture is based on the input/output relationships of the matched model of the finger and the model of the finger-tip.

3.2 An Architecture for Controlling the Grasping Wrench

This section presents the development of an architecture for controlling the grasping wrench based on the concept of impedance matching. Section 3.2.1 presents the open-loop architecture for controlling the grasping wrench in an ideal case when the exact model parameters of the finger and the finger-tip are known and there is no disturbance wrench acting on the finger. Section 3.2.2 presents the closed-loop architecture for controlling the grasping wrench based on the impedance matching concept and the impedance of the soft finger-tip and investigates the performance of the closed-loop architecture.

3.2.1 The Open-Loop Architecture

From *chapter 2*, the closed-loop *matched* dynamics of a finger can be written as:

$$W_{act}^{t} = \frac{1}{s}(\mathbf{Z}_t + \mathbf{Z}_u)T_{fe} \ , \tag{3.1}$$

where $\mathbf{Z}_t$ is the targeted impedance of a finger and $\mathbf{Z}_u$ is the impedance of the soft finger-tip, or:

$$W_{act}^{t} = \frac{1}{s}(\mathbf{Z}_s^*)T_{fe} \ ,$$

where $\mathbf{Z}_s^*$ is referred to as a known resultant impedance. Figure 3.1 shows a one-dimensional model of a known resultant impedance of a finger with the model of the soft finger-tip in contact with an object. From *assumption A.2* and *proposition 2.3* we have the following input/output relationship using the impedance model of the soft finger-tip, as:

$$W_g = \frac{1}{s}\mathbf{Z}_u T_{fe} \ . \tag{3.2}$$

In grasping, each finger of a dexterous mechanical hand must exert a wrench on the grasped object which is equal to the desired grasping wrench W_g^*. In general, the desired grasping wrench is determined by a grasp planner. As it was defined before, *section 2.3*, wrench is a through variable, *definition B.2*, in the spring and damper representations of the soft finger-tip. The desired grasping wrench W_g^* which a finger-tip should exert on the grasped object is determined by the finger end-point having a twist T_{fe}^*, i.e. an across variable, *definition B.3*, which is equal to the following

$$T_{fe}^* = s\mathbf{Z}_u^{-1} W_g^* \ . \tag{3.3}$$

In order to cause the finger end-point to have the desired twist T_{fe}^* defined by equation 3.3, the finger end-point actuator has to generate a ***through*** variable which is a function of the resultant impedance of a finger with soft finger-tip in contact with the grasped object $\mathbf{Z}_s^*$ or

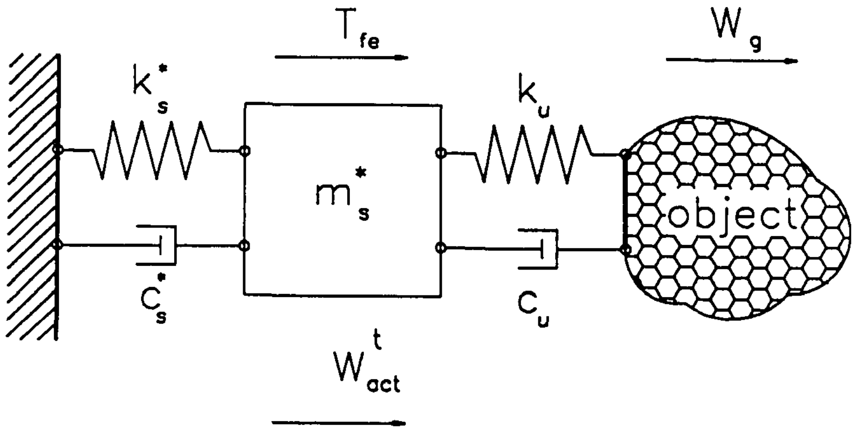


Figure 3.1 – A known resultant impedance model of a finger with soft finger–tip in contact with the grasped object.

$$W^*_{act} = W^t_{act} = \frac{1}{s}\mathbf{Z}^*_s T^*_{fe} \ . \tag{3.4}$$

Figure 3.2 shows the *open-loop* control block diagram architecture of the grasping wrench. In this control block diagram, the input W^*_g into the block $s\mathbf{Z}_u^{-1}$ will result in the output T^*_{fe}, see equation 3.4. The output of this block becomes an input twist T^*_{fe} into the block $\frac{1}{s}\mathbf{Z}^*_s$ which results in the desired finger end-point actuating wrench W^*_{act}. The desired finger end-point actuating wrench is then implemented by the resultant closed-loop admittance block $s[\mathbf{Z}^*_s]^{-1}$ of a finger to cause the finger end-point twist T_{fe}. This twist will become an input to the impedance block of the soft finger-tip $\frac{1}{s}\mathbf{Z}_u$ which will result in the grasping wrench W_g .

The *open-loop* relationship between the actual and the desired grasping wrench can be written as:

$$W_g = [\mathbf{Z}_u]^{-1}[\mathbf{Z}^*_s][\mathbf{Z}^*_s]^{-1}[\mathbf{Z}_u]\, W^*_g \ , \tag{3.5}$$

$$W_g = \mathbf{I}W^*_g \ .$$

The above equation states that in an ideal case when the exact model parameters of the finger and the finger-tip are known and there is no disturbance wrench acting on the finger, the open-loop control architecture results in the actual grasping wrench W_g to be equal to the desired grasping wrench W^*_g.

3.2.2 The Closed-Loop Architecture

In the presence of uncertainties in the actual parameters of the impedance model of the actual finger $\mathbf{Z}_f$, e.g. see equation 2.1, and the presence of the unwanted disturbance wrenches W_d which can act on the finger/object system, the actual grasping wrench W_g may not be equal to the desired grasping wrench W^*_g.

For example, for a 2*DOF* finger (Figure 3.3), let the transfer function between the

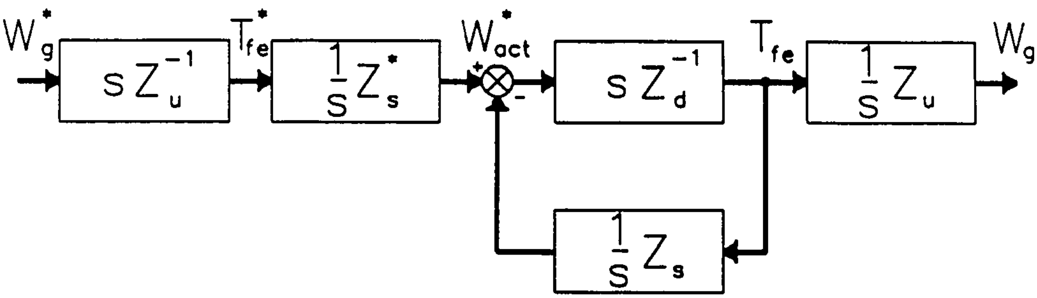


Figure 3.2 – The open-loop control architecture for controlling the grasping wrench of a finger.

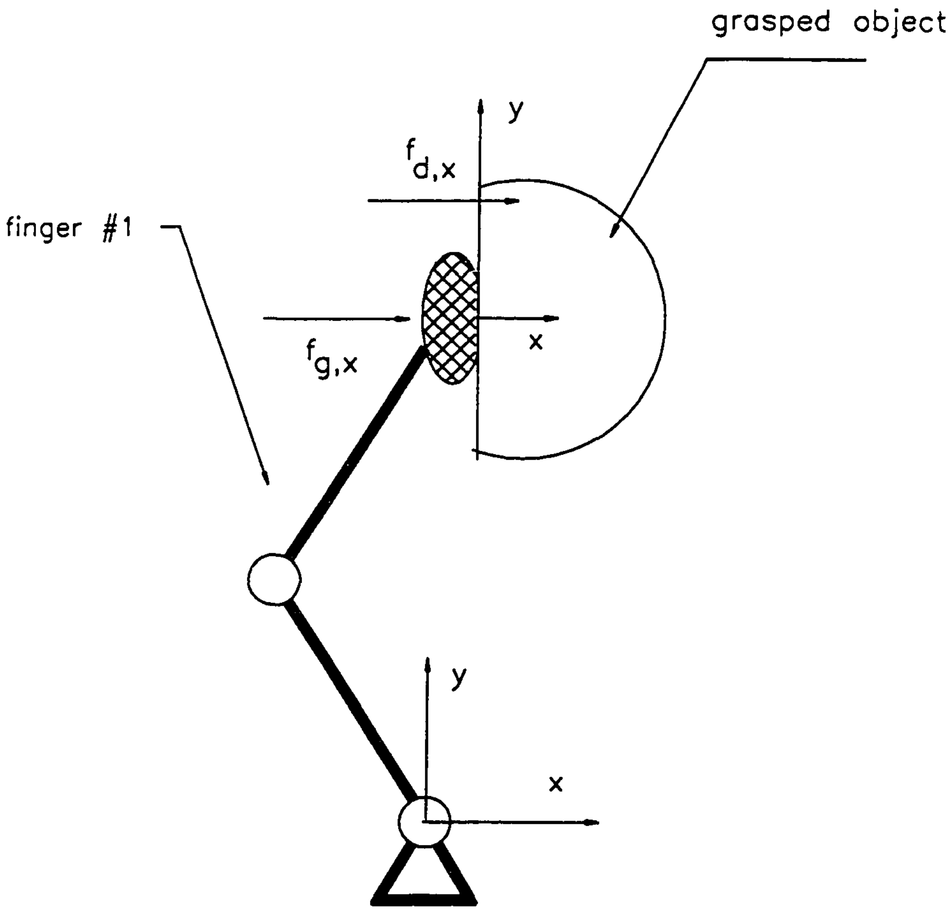


Figure 3.3 – A configuration of a finger in contact with the grasped object.

component of the grasping wrench along the x-direction of the finger end-point and the component of the disturbance wrench, when $f^*_{g,x}$=0 and all the parameters are known be written as:

$$\frac{f_{g,x}}{f_{d,x}} = \frac{c_{u,x}s+k_{u,x}}{m_{t,x}s^2+(c_{t,x}+c_{u,x})s+(k_{t,x}+k_{u,x})} \quad . \tag{3.6}$$

Using the final value theorem, Van de Vegte[27], the steady-state value of the component of the grasping wrench to the step input disturbance wrench can be obtained as:

$$f_{g,x} = \lim_{s \to 0} s\, f_{d,x}(s)\, Tr_x(s) = \frac{k_{u,x}}{k_{t,x}+k_{u,x}} \quad . \tag{3.7}$$

The above equation states that the steady-state value of the component of the grasping wrench $f_{g,x}$ to the step input disturbance is not equal to zero, i.e. $f_{g,x} \neq f^*_{g,x}$.

By comparing these wrenches, i.e. regulating the grasping wrench, an additional finger end-point actuating wrench can be generated which compensates for any differences. Figure 3.4 shows the closed-loop grasping wrench control architecture.

For a *2DOF* finger, Let $\tilde{\mathbf{Z}}_d = \varepsilon \mathbf{Z}_d$ represent the actual feedback decoupled impedance model of a finger where, for example, comparing with the $\mathbf{Z}_d$, there are differences in their diagonal elements. Let $\tilde{\mathbf{Z}}_u$ represent the approximate model of the soft finger tip. The transfer function of the control architecture of Figure 3.4 along the x-direction of the finger end-point when W_d=0 can be written as:

$$\frac{f_{g,x}}{f^*_{g,x}} = \tilde{Tr}_x(s) = \frac{D(s)}{N(s)} \quad , \tag{3.8}$$

where:

$$D(s) = m_{t,x}c_{u,x}s^3+(m_{t,x}k_{u,x}+(c_{t,x}+\tilde{c}_{u,x})c_{u,x})s^2+$$

$$((c_{t,x}+\tilde{c}_{u,x})k_{u,x}+(k_{t,x}+\tilde{k}_{u,x})c_{u,x})s+(k_{t,x}+\tilde{k}_{u,x})k_{u,x} \quad ,$$

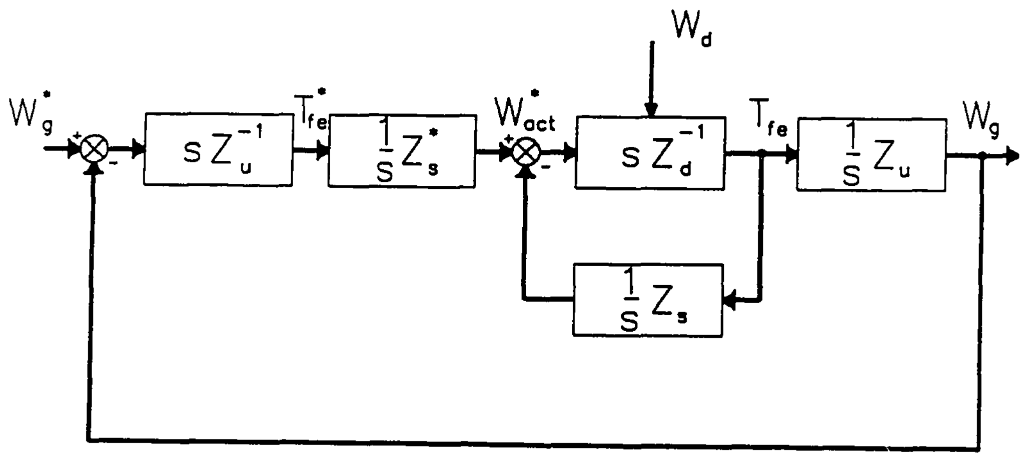


Figure 3.4 – The closed-loop control architecture for controlling the grasping wrench of a finger.

$$N(s) = (\tilde{c}_{u,x}(m_{t,x}-1+\varepsilon)+m_{t,x}c_{u,x})s^3+(\tilde{c}_{u,x}(c_{t,x}+\tilde{c}_{u,x})+\tilde{k}_{u,x}(m_{t,x}-1+\varepsilon)+$$

$$(c_{t,x}+\tilde{c}_{u,x})c_{u,x}+m_{t,x}k_{u,x})s^2+(\tilde{c}_{u,x}(k_{t,x}+k_{u,x})+\tilde{k}_{u,x}(c_{t,x}+\tilde{c}_{u,x})+$$

$$c_{u,x}(k_{t,x}+\tilde{k}_{u,x})+(c_{t,x}+\tilde{c}_{u,x})k_{u,x})s+\tilde{k}_{u,x}(k_{t,x}+k_{u,x})+(k_{t,x}+\tilde{k}_{u,x})k_{u,x}) \ ,$$

where ε represent the magnitude of the diagonal elements of $\tilde{\mathbf{Z}}_d$ and $\tilde{k}_{u,x}$ and $\tilde{c}_{u,x}$ represent the approximate model of the soft finger-tip. Using the final value theorem, the steady-state value of the component of the grasping wrench along the x-direction of the finger end-point $f_{g,x}$, to the step input in $f^*_{g,x}$ can be written as:

$$f_{g,x}=\lim_{s \to 0} s \ f^*_{g,x}(s)\,\tilde{T}r_x(s) \ , \tag{3.9}$$

or:

$$f_{g,x} = \frac{(k_{t,x}+\tilde{k}_{u,x})k_{u,x}}{\tilde{k}_{u,x}(k_{t,x}+k_{u,x})+(k_{t,x}+\tilde{k}_{u,x})k_{u,x}} \ .$$

As can be seen from the above, the steady-state magnitude of the $f_{g,x}$ does not approach one even when $\tilde{k}_{u,x}=k_{u,x}$, hence, the closed-loop controller dose not have the desired performance.

To further examine the closed-loop performance of the control architecture of Figure 3.4, let us assume that there is a constant disturbance wrench acting along the x-direction of the finger end-point and there are some uncertainties in the model parameters of the finger and the soft finger-tip. The steady-state response of the component of the grasping wrench $f_{g,x}$ to the step input disturbance wrench $f_{d,x}$ when $f^*_{g,x}=0$ can be written as:

$$f_{g,x} = \frac{\tilde{k}_{u,x}k_{u,x}}{k_{u,x}k_{t,x}+\tilde{k}_{u,x}k_{u,x}+\tilde{k}_{u,x}k_{t,x}+\tilde{k}_{u,x}\tilde{k}_{u,x}} \ . \tag{3.10}$$

As can be seen the steady-state value never approaches zero even when the exact parameters of the soft finger-tip are known, i.e. $\tilde{k}_{u,x}=k_{u,x}$.

The remainder of the chapter will present an alternative definition for the feedforward admittance and impedance blocks of the general controller such that the controller has the desired performance. The modified architecture has the property that error between the actual and the desired grasping wrench asymptotically approaches zero in the presence of constant disturbance wrench and variations in the model parameters of the finger, see section B.4.

3.3 A Nominal Model of a Finger with Soft Finger-Tip

This section presents formulation of the closed-loop dynamics model of a finger with soft finger-tip in contact with the grasped object. This dynamic model is referred to as a *nominal* model which is then used in section 3.5 to examine the existence of a robust controller for controlling the grasping wrench. This model is obtained by rewriting the control law for implementing the impedance matching concept introduced in the previous chapter.

Definition 3.1: A *nominal* model of a finger with soft finger-tip in contact with the grasped object is obtained by assuming that the actual dynamic parameters of the finger and the soft finger-tip are known and there is no disturbance wrench acting on the finger/object system.

The simplified linear dynamic model of a *2DOF* planar finger, see *remark D.2,* in contact with the grasped object is written as:

$$\mathbf{M}_{fe}\dot{T}_{fe} = W_{act} - W_g \ . \tag{3.11}$$

From equation 2.10, a linear dynamic decoupling control law is written as:

$$W_{act} = \mathbf{M}_{fe} W_{act}^{e} + W_g \ , \tag{3.12}$$

where the grasping wrench W_g is calculated from the impedance model of the soft finger-tip given in equation 3.2. Substituting the control law of equation 3.12 into the dynamic equation 3.11 we have:

$$\dot{T}_{fe} = W^e_{act} \quad . \tag{3.13}$$

Equation 3.13 represents a decoupled dynamic model of a finger with soft finger-tip in contact with the grasped object.

From equation 2.13, the matching condition can be written as:

$$\mathbf{Z}_s = \mathbf{Z}_t + \mathbf{Z}_u - \mathbf{Z}_d \quad .$$

From equations 3.13 and 2.12 we have $\mathbf{Z}_d = s^2\mathbf{I}$. Therefore, by considering a model of the targeted impedance $\mathbf{Z}_t$ and a model of soft finger-tip $\mathbf{Z}_u$ the above equation can be written as:

$$\mathbf{Z}_s = (\mathbf{M}_t - \mathbf{I})s^2 + (\mathbf{C}_t + \mathbf{C}_u)s + (\mathbf{K}_t + \mathbf{K}_u) \quad , \tag{3.13a}$$

where $\mathbf{M}_t - \mathbf{I}$ is the gain matrix for the derivative of the twist of the finger end-point, i.e. $\dot{T}_{fe}$.

Figure 3.5 shows the equivalent control block diagram of implementation of impedance matching of figure 2.3. Including the feedback gain on the derivative of the finger end-point twist in the linear dynamic decoupling control law of equation 3.12, we have:

$$W_{act} = \mathbf{M}_{fe}(W^e_{act} - (\mathbf{M}_t - \mathbf{I})\dot{T}_{fe}) + W_g \quad . \tag{3.14}$$

Substituting the control law of equation 3.14 into equation 3.11 we obtain the following :

$$\mathbf{M}_{fe}\dot{T}_{fe} = \mathbf{M}_{fe}W^e_{act} - \mathbf{M}_{fe}\mathbf{M}_t\dot{T}_{fe} + \mathbf{M}_{fe}\dot{T}_{fe} + W_g - W_g \quad . \tag{3.15}$$

Simplifying the above, the following dynamic model of finger with soft finger-tip in

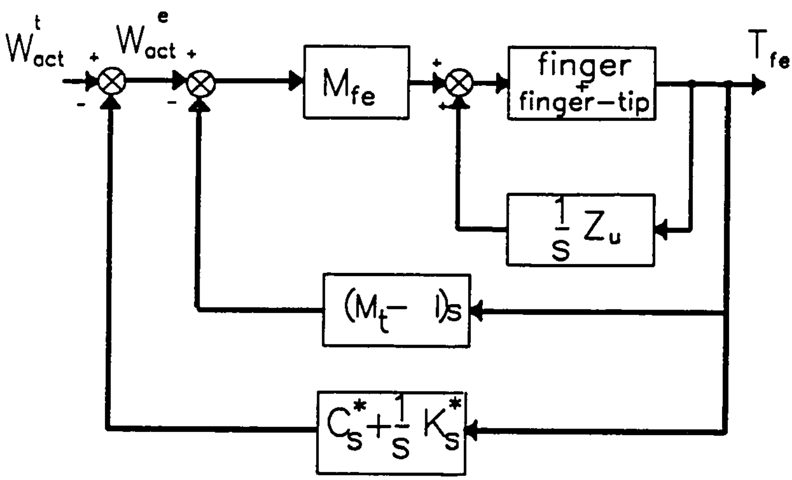


Figure 3.5 – The equivalent implementation of the impedance matching concept.

contact with the grasped object is obtained as:

$$W^e_{act} = \frac{1}{s}(\mathbf{M}_t s^2)T_{fe} = \frac{1}{s}\mathbf{Z}^m_d T_{fe} \ , \tag{3.16}$$

where $\mathbf{Z}^m_d = \mathbf{M}_t s^2$. The impedance matching condition $\mathbf{Z}_s$ of equation 2.13 is then rewritten in the modified form which includes only the gain matrices on the finger end-point twist and its integral : (Figure 3.6)

$$\mathbf{Z}^m_s = (\mathbf{C}_t + \mathbf{C}_u)s + (\mathbf{K}_t + \mathbf{K}_u) \ . \tag{3.17}$$

In the standard state-space form, equation 3.16 is written as:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \ , \tag{3.18}$$

where:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (m_{t,x})^{-1} & 0 \\ 0 & (m_{t,y})^{-1} \end{bmatrix} ,$$

and $x \in \mathbf{R}^4$ is defined as $x = (\int v_{fe,x}dt, \int v_{fe,y}dt, v_{fe,x}, v_{fe,y})^T$ and $u \in \mathbf{R}^2$ is defined as $u = W^e_{act} = (f^e_{act,x}, f^e_{act,y})^T$.

The output vector $y = W_g = (f_{g,x}, f_{g,y})^T$ as a function of the state of the system is written as:

$$y = W_g = \mathbf{C}x \ , \tag{3.19}$$

where :

$$\mathbf{C} = \begin{bmatrix} k_{u,x} & 0 & c_{u,x} & 0 \\ 0 & k_{u,y} & 0 & c_{u,y} \end{bmatrix} \quad \text{and} \quad y \in \mathbf{R}^2 \ .$$

Equations 3.18-3.19 represent the *nominal* model of a finger with soft finger-tip in

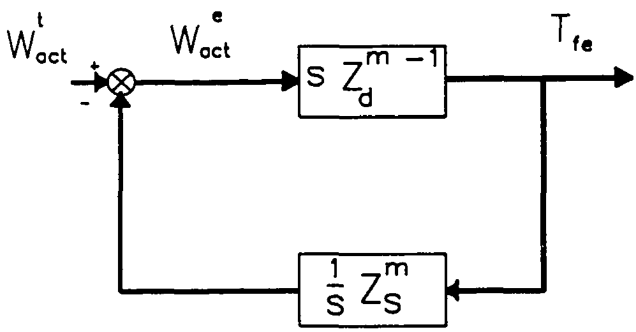


Figure 3.6 - The modified feedback decoupled and the modified matching condition.

contact with the grasped object.

3.4 Sources of Uncertainties in the Nominal Model

The previous section presented the formulation of the nominal model of a $2DOF$ finger. The underlying assumption, i.e. *assumption 2.2*, in the above derivation was that the exact knowledge of the dynamic parameters of a finger and the soft finger-tip were available and there was no disturbance wrench acting on the finger/object system.

This section presents the formulation of the modified feedback decoupled dynamic model of a finger with soft finger-tip where the *assumption 2.2* does not hold.

In practical implementation, the linear dynamic decoupling control law of equation 3.14 is written as:

$$W_{act} = \tilde{\mathbf{M}}_{fe}(W^{e}_{act} - \mathbf{M}_s \dot{T}_{fe}) + \tilde{W}_g \ , \tag{3.20}$$

where $\mathbf{M}_s = \mathbf{M}_t - \mathbf{I}$ and $\tilde{\mathbf{M}}_{fe}$ represent the approximate inertia matrix of a finger expressed in finger end-point coordinate frame. Also, since the grasping wrench is calculated from the knowledge of soft finger-tip impedance and the state of the finger end-point, $\tilde{W}_g$ represents the calculated value of the grasping wrench based on the approximate model of the soft finger-tip given as:

$$\tilde{W}_g = \frac{1}{s}\tilde{\mathbf{Z}}_u T_{fe} \ , \tag{3.21}$$

where $\tilde{\mathbf{Z}}_u$ is the approximate impedance model of the soft finger-tip or:

$$\tilde{\mathbf{Z}}_u = \tilde{\mathbf{C}}_u s + \tilde{\mathbf{K}}_u \ . \tag{3.22}$$

The actual linear dynamic model of a finger, equation 3.11, which includes the presence of the disturbance wrench W_d, is written as :

$$\mathbf{M}_{fe}\dot{T}_{fe} = W_{act} - W_g + W_d \ , \tag{3.23}$$

where $\mathbf{M}_{fe}$ is the actual inertia matrix of a finger, W_g is the actual grasping wrench and W_d is a representation of the disturbance wrench.

Assumption 3.1: The *disturbance wrench* W_d is due to any unwanted collision of the finger/object system with the moving obstacles in the work-cell or due to the effects which are not included in the dynamic model of a finger, see equation 3.11. These effects can be due to the coulomb friction wrenches at each joint of a finger.

Substituting the control law of the equation 3.20 into the dynamic equation 3.23 we have:

$$\mathbf{M}_{fe}\dot{T}_{fe} = \tilde{\mathbf{M}}_{fe}W^e_{act} - \tilde{\mathbf{M}}_{fe}\mathbf{M}_s\dot{T}_{fe} + \tilde{W}_g - W_g + W_d \ . \qquad (3.24)$$

Rewriting the above equation as:

$$\dot{T}_{fe} = [\mathbf{M}_{fe}+\tilde{\mathbf{M}}_{fe}\mathbf{M}_s]^{-1}\tilde{\mathbf{M}}_{fe}W^e_{act} + [\mathbf{M}_{fe}+\tilde{\mathbf{M}}_{fe}\mathbf{M}_s]^{-1}(\tilde{W}_g-W_g)$$
$$+ [\mathbf{M}_{fe}+\tilde{\mathbf{M}}_{fe}\mathbf{M}_s]^{-1}W_d \ . \qquad (3.25)$$

Equation 3.25 represents the implementation of equation 3.14 where the exact knowledge of the dynamic parameters of a finger and its soft finger-tip are not available. Rewriting the above equation in state-space representation we have:

$$\dot{x} = \begin{bmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} (\tilde{W}_g-W_g) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} W_d \ , \qquad (3.26)$$

where :

$$[\mathbf{M}_{fe}+\tilde{\mathbf{M}}_{fe}\mathbf{M}_s]^{-1}\tilde{\mathbf{M}}_{fe} = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} ,$$

$$[\mathbf{M}_{fe}+\tilde{\mathbf{M}}_{fe}\mathbf{M}_s]^{-1} = \begin{bmatrix} e_1 & e_2 \\ e_3 & e_4 \end{bmatrix} .$$

The output of the system is the grasping wrench $\tilde{W}_g$ which is based on the approximate model of the finger-tip is written as:

$$\tilde{W}_g = y = \begin{bmatrix} \tilde{k}_{u,x} & 0 & \tilde{c}_{u,x} & 0 \\ 0 & \tilde{k}_{u,y} & 0 & \tilde{c}_{u,y} \end{bmatrix} x \ . \tag{3.27}$$

Equations 3.26-3.27 can be written as:

$$\dot{x} = \mathbf{A}x + \tilde{\mathbf{B}}u + \mathbf{E}_1(\tilde{W}_g - W_g) + \mathbf{E}_1\, W_d \ , \tag{3.28}$$

$$y = \tilde{\mathbf{C}}\, x \ . \tag{3.29}$$

In equation 3.28, two types of disturbance terms are presented, namely, $\mathbf{E}_1(\tilde{W}_g - W_g)$ and $\mathbf{E}_1 W_d$.

Definition 3.2: *Non-exogenous* disturbance is defined as the type of disturbance which depends upon the state of the system, i.e. a state of a finger of a dexterous mechanical hand x.

In equation 3.28, the *non-exogenous* disturbance arises from the inexact cancellation of the actual W_g and the calculated $\tilde{W}_g$ grasping wrench. However, the effect of this inexact cancellation can be represented as a change in the system matrix **A**. Rewriting equation 3.28 we have:

$$\dot{x} = \mathbf{A}x + \tilde{\mathbf{B}}u + \mathbf{E}_1[\tilde{\mathbf{C}} - \mathbf{C}]x + \mathbf{E}_1 W_d \ .$$

In its simplified form as:

$$\dot{x} = [\mathbf{A} + \mathbf{E}_1[\tilde{\mathbf{C}} - \mathbf{C}]]x + \tilde{\mathbf{B}}u + \mathbf{E}_1 W_d \ ,$$

$$or \quad \dot{x} = \tilde{\mathbf{A}}x + \tilde{\mathbf{B}}u + \mathbf{E}_1 W_d \ . \tag{3.30}$$

The other type of disturbance $\mathbf{E}_1 W_d$ which appears in equation 3.28 is the *exogenous* disturbance wrench.

Definition 3.3: *Exogenous* disturbance is defined as the type of disturbance which does not depend on the state of the system, i.e. state of a finger of a dexterous mechanical hand x.

From *assumption 3.1,* this type of disturbance arises from, for example, the unwanted collision of the finger/object system with the moving obstacle in the work-cell of a dexterous mechanical hand or the effect of disturbances due to the presence of coulomb friction wrenches in each joint of a finger.

Comparing equations 3.29-3.30 with the nominal model of a finger with soft finger-tip in contact with the grasped object, see equations 3.18-3.19, it is obvious that in practical implementation, there are differences between the actual model and the nominal model of a finger. For example, $\tilde{\mathbf{A}} \neq \mathbf{A}$, $\tilde{\mathbf{B}} \neq \mathbf{B}$,$\tilde{\mathbf{C}} \neq \mathbf{C}$ and the presence of unwanted disturbance .

In general, if a control law is designed based on the *nominal* model of a system to have certain performance specifications, its actual implementation may not have the desired performance. However, if the control law is robust, its actual implementation will have similar properties as of the desired *nominal* performance.

3.5 Existence of a Robust Controller

This subsection, based on the results of *theorem B.2,* Davison, Goldenberg[24], examines the conditions for existence of robust controller given the *nominal* model of the system, see equations 3.18-3.19.

The first condition that a nominal system must satisfy for existence of a robust controller is that the system should be *stabilizable* see *definition B.11.*

From the nominal system descriptions of a *2DOF* finger, we have the following system-parameter matrices:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ (m_{t,x})^{-1} & 0 \\ 0 & (m_{t,y})^{-1} \end{bmatrix} ; \mathbf{C} = \begin{bmatrix} k_{u,x} & 0 & c_{u,x} & 0 \\ 0 & k_{u,y} & 0 & c_{u,y} \end{bmatrix} . \quad (3.31)$$

From the above and equation B.15 we have $n=4$, $r=2$, $m=2$. The condition of *stabilizability* based on the equation B.15 and the system-parameter matrices of equation 3.31 is written as:

$$rank \begin{bmatrix} 0 & 0 & (m_{t,x})^{-1} & 0 & 0\;0\;0\;0 \\ 0 & 0 & 0 & (m_{t,y})^{-1} & 0\;0\;0\;0 \\ (m_{t,x})^{-1} & 0 & 0 & 0 & 0\;0\;0\;0 \\ 0 & (m_{t,y})^{-1} & 0 & 0 & 0\;0\;0\;0 \end{bmatrix} = n = 4 \ . \quad (3.32)$$

The result of the above condition states that the nominal system is *stabilizable.*

The second condition of the *theorem B.2* is that the nominal system should be *observable, see definition B.12.* From equation B.16 and the system-parameter matrices of equation 3.31 we have:

$$rank \begin{bmatrix} k_{u,x} & 0 & c_{u,x} & 0 \\ 0 & k_{u,y} & 0 & c_{u,y} \\ 0 & 0 & k_{u,x} & 0 \\ 0 & 0 & 0 & k_{u,y} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = n = 4 \ . \quad (3.33)$$

The above condition states that the nominal system is *observable.*

The third condition is that the number of the control inputs must be equal to or greater than the number of the regulated outputs. For the nominal system representation we have ($m = n = 2$).

The condition of equation B.17 for the nominal system-parameter given in equation 3.31 is written as:

$$\begin{bmatrix} -\nu & 0 & 1 & 0 & 0 & 0 \\ 0 & -\nu & 0 & 1 & 0 & 0 \\ 0 & 0 & -\nu & 0 & (m_{t,x})^{-1} & 0 \\ 0 & 0 & 0 & -\nu & 0 & (m_{t,y})^{-1} \\ k_{u,x} & 0 & c_{u,x} & 0 & 0 & 0 \\ 0 & k_{u,y} & 0 & c_{u,y} & 0 & 0 \end{bmatrix}, \tag{3.34}$$

where ν is set equal to the roots of the characteristic equations of the linear differential equations modelling the exogenous inputs. For a given model of the exogenous input, see equation B.19, as long as the rank of the matrix given by equation 3.34 is equal to $n+min(r,m)$ or for the example of this chapter, $rank=6$, a robust controller exists. Also, the output of the system can be measured using the force measuring transducer or can be calculated using the knowledge of the soft finger-tip material properties as a function of the finger end-point twist.

Analogous to the above condition, we note that the nominal model represents a set of single-input/single-output transfer functions between the output grasping wrench W_g and the input actuating wrench W^e_{act} defined by:

$$W_g = [\mathbf{Z}^m_d]^{-1}\mathbf{Z}_u \, W^e_{act} \,.$$

In the above equation it can be seen that the location of zero of a decoupled transfer function, i.e. transmission zero, is defined by the model of the finger-tip. Therefore, from *theorem B.2* the roots of characteristic equation of exogenous disturbance wrench should not be equal to the zeros of *SISO* transfer function matrices, i.e. no pole zero cancellation.

The above results conclude that there exists a robust controller for the nominal system defined in equations 3.18-3.19 with the system-parameter matrices specified in equation 3.31 as long as for a given model of exogenous inputs the rank condition of equation 3.34 is satisfied.

3.6 Robust Architecture for Controlling the Grasping Wrench

Previous sections defined a modified feedback decoupled dynamic model of a finger with soft finger-tip in contact with the grasped object and demonstrated a need for a robust controller. Also, based on the conditions of *theorem B.2* it was shown that the nominal system satisfies the conditions for existence of a robust controller as long as the roots of characteristic equation which model the exogenous disturbances satisfy the rank condition of equation 3.34.

This section shows how the definitions of the admittance/impedance blocks in the feedforward path of a general architecture for controlling the grasping wrench are modified so the new controller is robust to the constant exogenous inputs.

In the actual dynamic model of a finger with a soft finger-tip in contact with the grasped object, see equation 3.23, it was assumed that W_d models the *exogenous* disturbance arising from the collision of the finger/object system with the unwanted moving obstacle in the work-cell or the effect of friction in each joint of a finger. The model of this type of disturbance wrench along the finger end-point reference coordinate frame is presented as a first order linear differential equation given by

$$\dot{W}_d = 0 \quad or \quad W_d = c \ , \tag{3.35}$$

where c is scalar. The roots of characteristic equations of the exogenous disturbance along the finger end-point reference coordinate frame defined in equation 3.35 are all equal to zero, i.e. $s_{1,x}=0, s_{1,y}=0$. Substituting for $\nu = s_{1,x}$ and $\nu = s_{1,y}$ into rank condition of equation 3.34 it can be seen that the rank of the matrix is equal to 6, therefore, a robust controller exists.

From the control block diagram of Figure 3.4 and the general model of the servo-compensator defined in *appendix B*, see equation B.21, the above equation can be written as:

$$T^*_{fe} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} (\int T^*_{fe} dt) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \end{Bmatrix} , \qquad (3.36)$$

or :

$$\dot{\xi} = \Lambda \xi + \beta e \ .$$

For this example, the gain matrix on the state of the servo-compensator ξ defined by equation B.20 is written as:

$$\mathbf{K}_{ser} = \begin{bmatrix} k_{ser,x} & 0 \\ 0 & k_{ser,y} \end{bmatrix} . \qquad (3.37)$$

The stabilizing gain matrix defined in equation B.20 are obtained from the modified matching condition given in equation 3.17, or

$$\mathbf{K}_{sta} = \begin{bmatrix} k_{t,x}+k_{u,x} & 0 & c_{t,x}+c_{u,x} & 0 \\ 0 & k_{t,y}+k_{u,y} & 0 & c_{t,y}+c_{u,y} \end{bmatrix} . \qquad (3.38)$$

Based on the theory, the gain parameters must be selected such that the closed-loop system is stable.

The controller of equation B.20 with the gain matrices defined in equations 3.37 and 3.38 represents a decoupled robust controller for a decoupled nominal model of a finger with soft finger-tip. The relationship between the actual grasping wrench and the desired one can be written as: (see Figure 3.7)

$$W_g = [[s\mathbf{I}]^{-1} \mathbf{K}_{ser} [\mathbf{Z}^*_s]^{-1} \mathbf{Z}_u][\mathbf{I} + [s\mathbf{I}]^{-1} \mathbf{K}_{ser} [\mathbf{Z}^*_s]^{-1} \mathbf{Z}_u]^{-1} \ W^*_g \ . \qquad (3.39)$$

Let the *SISO* transfer function of the component of the grasping wrench, e.g. $f_{g,x}$, along the x-direction of the finger end-point reference coordinate frame system be written as:

$$\frac{f_{g,x}}{f^*_{g,x}} = \frac{k_{ser,x} c_{u,x} s + k_{ser,x} k_{u,x}}{s((m_{t,x} - 1 + 1)s^2 + (c_{t,x} + c_{u,x})s + (k_{t,x} + k_{u,x})) + k_{ser,x}(c_{u,x} s + k_{u,x})} = Tr_x(s) \quad (3.40)$$

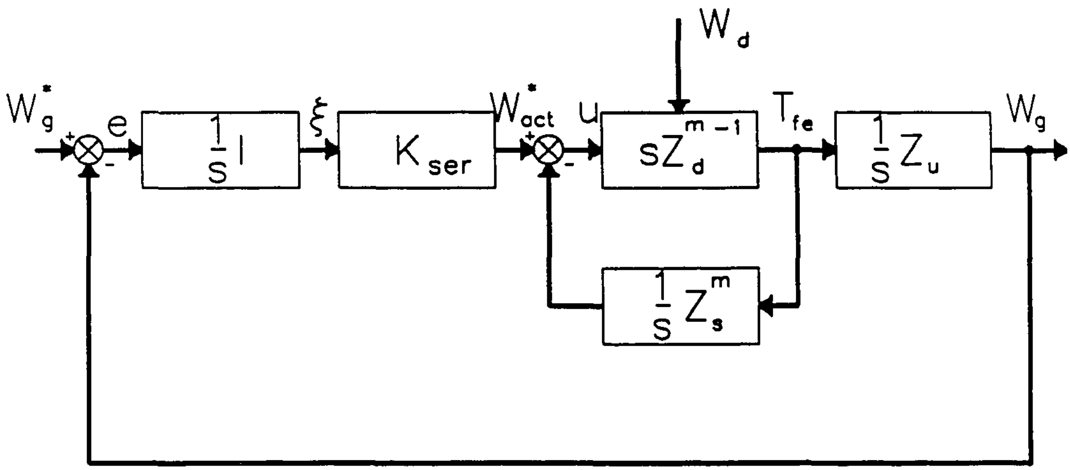


Figure 3.7 – The robust grasping wrench controller of a finger.

From the *Routh-Hurwitz* criterion, Van de Vegte[27], the stability of the above closed-loop transfer function is achieved if the following conditions are satisfied:

$$m_{t,x}>0 \ ; \ c_{t,x}+c_{u,x}>0 \ ; \ k_{ser,x}k_{u,x}>0 \ ;$$

$$k_{t,x}+k_{u,x}+k_{ser,x}c_{u,x}>\frac{m_{t,x}k_{ser,x}k_{u,x}}{c_{t,x}+c_{u,x}} \ . \qquad (3.41)$$

Similar constraint relationships can also be obtained along the y-direction of the finger end-point reference coordinate frame.

Let us assume that there are some uncertainties in the diagonal elements of the feedback decoupled impedance model of a finger with soft finger-tip, i.e $\tilde{\mathbf{Z}}_d = \varepsilon \mathbf{Z}_d$ and in the knowledge of the impedance representation of the soft finger-tip. The *SISO* transfer function of equation 3.40 is written as:

$$\frac{f_{g,x}}{f^*_{g,x}} = \tilde{T}r_x(s) =$$

$$\frac{k_{ser,x}c_{u,x}s+k_{ser,x}k_{u,x}}{(m_{t,x}-1+\varepsilon)s^3+(c_{t,x}+\tilde{c}_{u,x})s^2+((k_{t,x}+\tilde{k}_{u,x})+k_{ser,x}c_{u,x})s+k_{ser,x}k_{u,x}} \ , \qquad (3.42)$$

where $(\tilde{\ })$ represents the approximate values of the impedance model of the soft finger-tip and ε represents the actual magnitude of the diagonal element of the feedback decoupled model of a finger with soft finger-tip in contact with the grasped object.

Using the final value theorem, the steady-state error between the actual grasping wrench and the desired one is written as:

$$e_x = (f^*_{g,x}-f_{g,x}) = f^*_{g,x}-\lim_{s\to 0} s \, f^*_{g,x}(s)\tilde{T}r_x(s) = 0 \ . \qquad (3.43)$$

Equation 3.43 states that the output regulation always occurs between the actual output $f_{g,x}$ and the desired exogenous input $f^*_{g,x}$ even in the presence of uncertainties in the actual parameters of the finger and finger-tip.

To further demonstrate the property of the closed-loop controller using simplified examples, let the following represent the transfer function between the $f_{g,x}$ and the component of the disturbance wrench $f_{d,x}$ when $f^*_{g,x} = 0$ and also when there are some uncertainties in the model parameters:

$$\frac{f_{g,x}}{f_{d,x}} = \frac{s\,(c_{u,x}s + k_{u,x})}{(m_t - 1 + \varepsilon)s^3 + (c_{t,x} + \tilde{c}_{u,x})s^2 + (k_{t,x} + \tilde{k}_{u,x} + k_{ser,x}c_{u,x})s + k_{ser,x}k_{u,x}} = \tilde{Tr}_x(s)$$

Using the final value theorem, the steady-state value of the $f_{g,x}$ to the step input in $f_{d,x}$ can be written as:

$$f_{g,x} = \lim_{s \to 0} s\, f_{d,x}(s)\, \tilde{Tr}_x(s) = 0 \ .$$

As it can be seen from the above, the steady-state value of the $f_{g,x} = f^*_{g,x} = 0$ regardless of the disturbance. The next chapter will present the actual experimental performance of the robust grasping wrench controller for a $2DOF$ planar finger making soft contact with the rigid wall and exerting a desired wrench, i.e. W^*_g.

Figure 3.8 shows the simulated step response of the equation 3.40. This response is referred to as a nominal response where it is assumed that the exact parameters of the finger are known and there is no disturbance wrench acting on the finger. Figure 3.9 shows the step response of the equation 3.42 when $\tilde{\mathbf{Z}}_d = \varepsilon \mathbf{Z}_d$ where $\varepsilon = 0.8 \neq 1$. As can be seen from the figure, the actual response asymptotically follows the desired response. Figure 3.10 shows the step response of the controller when a disturbance wrench $f_{d,x}=1$ is acting on the finger. As can be seen from the figure, the response has a similar property as the response of Figure 3.8.

3.6.1 Algorithm to Design a Robust Controller for Constant Disturbance

This subsection outlines the necessary steps for designing a robust controller for grasping wrench based on the concept of impedance matching:

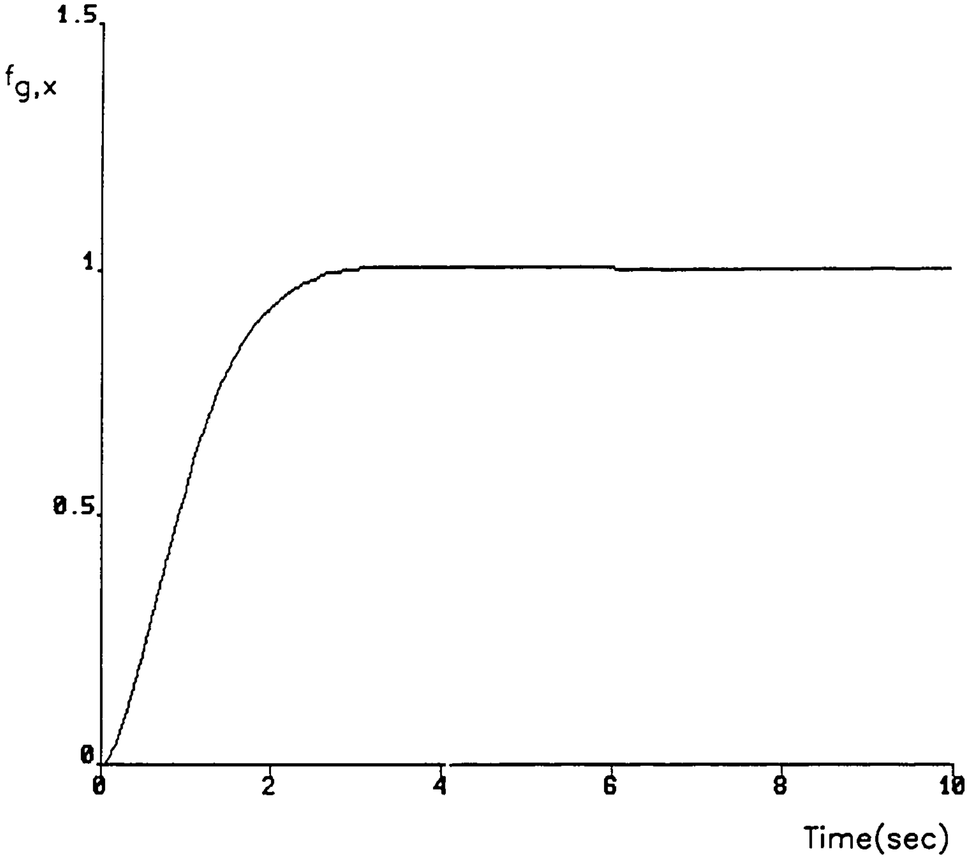


Figure 3.8 – A nominal response of the robust controller along the x-direction of the finger end-point.

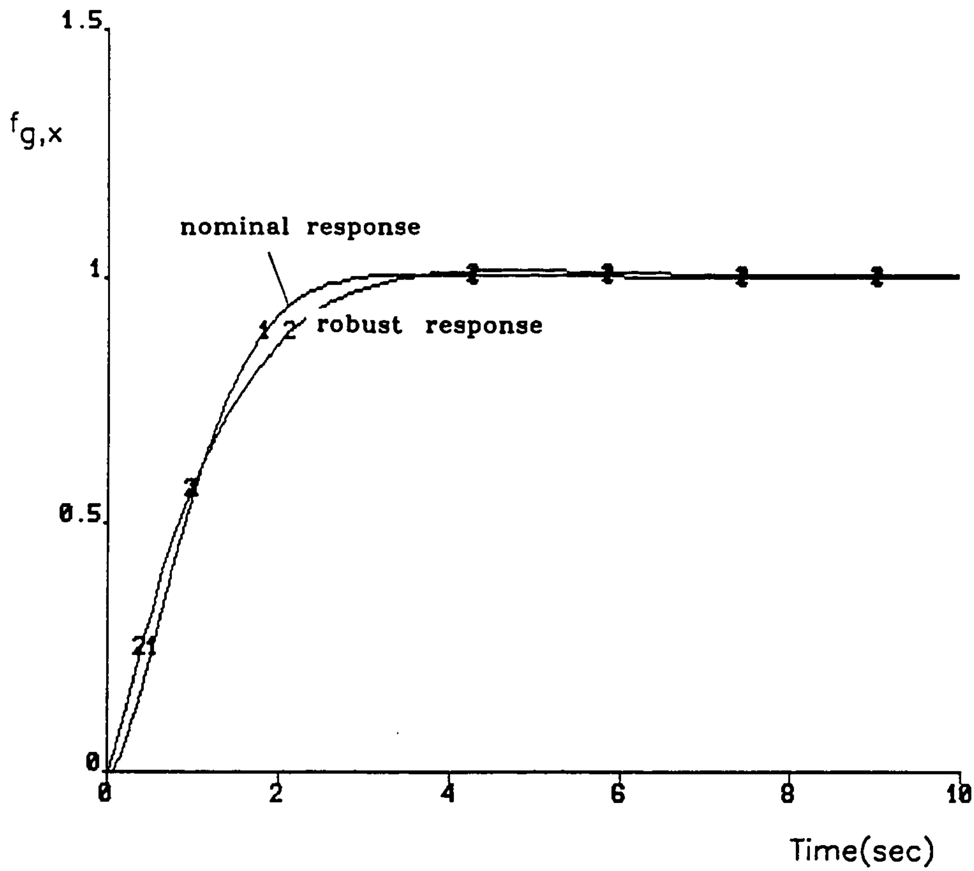


Figure 3.9 – A response of the robust controller when there is uncertainty in the system parameter.

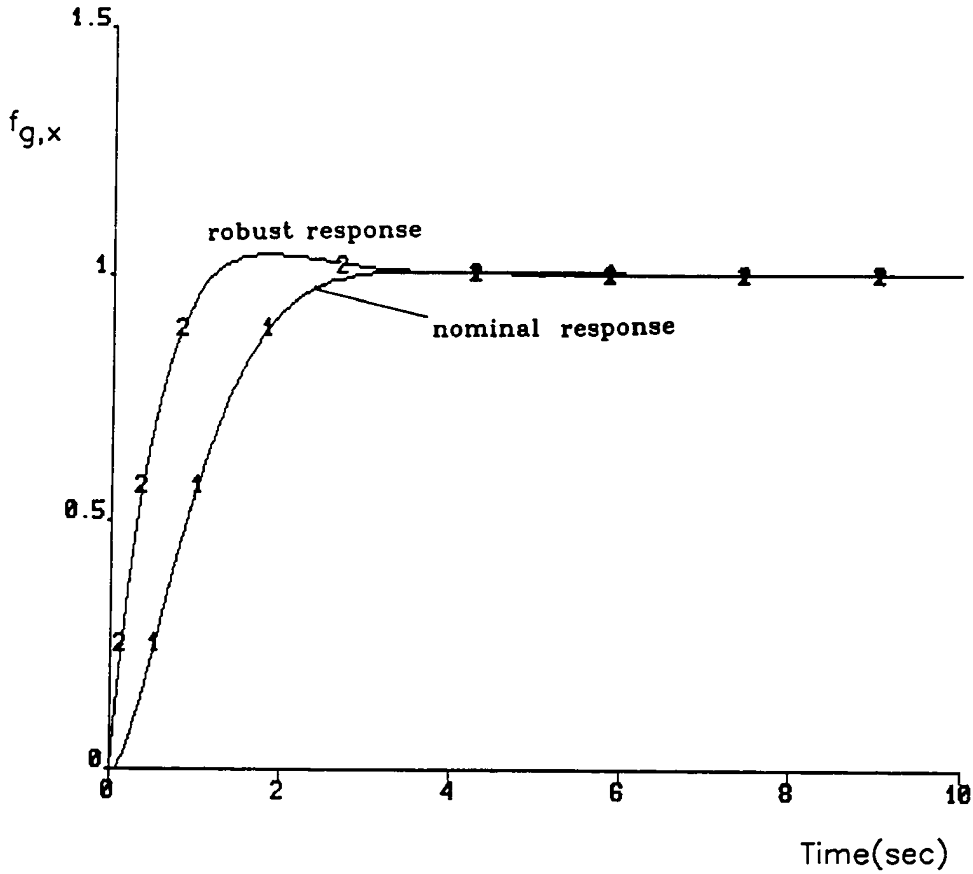


Figure 3.10 – A response of the robust controller when a disturbance wrench along the x–direction of the finger end–point is present.

Step 1: Determine the type of task which is assigned to the grasped object, e.g. a contact task.

Step 2: Determine an impedance model of the soft finger-tip.

Step 3: Determine a satisfactory range for the desired model parameters of the grasping finger, i.e. targeted impedance, see *appendix C*.

Step 4: Determine the gain matrix $\mathbf{K}_{ser}$ of the *servo-compensator* which results in the stable closed-loop system, i.e. inequalities 3.41.

3.7 An Application of the General Robust Controller to Tooling Tasks

This subsection presents an application of the solution to the general servomechanism problem outlined in the *section B.4,* to the control of the grasping wrench when a tooling task is assigned to the grasped object. A method for controlling the contacting wrench of the manipulator when it is performing a deburring operation was also proposed by Kazerooni[47]. This method is based on the impedance control concept which was proposed by Hogan[20].

Figure 3.11 shows a schematic of a tooling task. In this figure, the fingers of a dexterous mechanical hand have grasped a tool, e.g. cutting or abrasive tools, where the manipulator arm follows a specific trajectory with a specific feedrate velocity. As a result, the tool produces a desired finished surface regardless of the disturbance wrench which can act between the tool and the work-piece.

Assumption 3.2: Using a skilled operator equipped with a force sensing transducer the required wrench that he exerts while moving along the surface of the work-piece can be recorded, e.g. see Asada[29].

Assumption 3.3: The actual components of the disturbance wrench W_d along the finger end-point coordinate frame are given as:

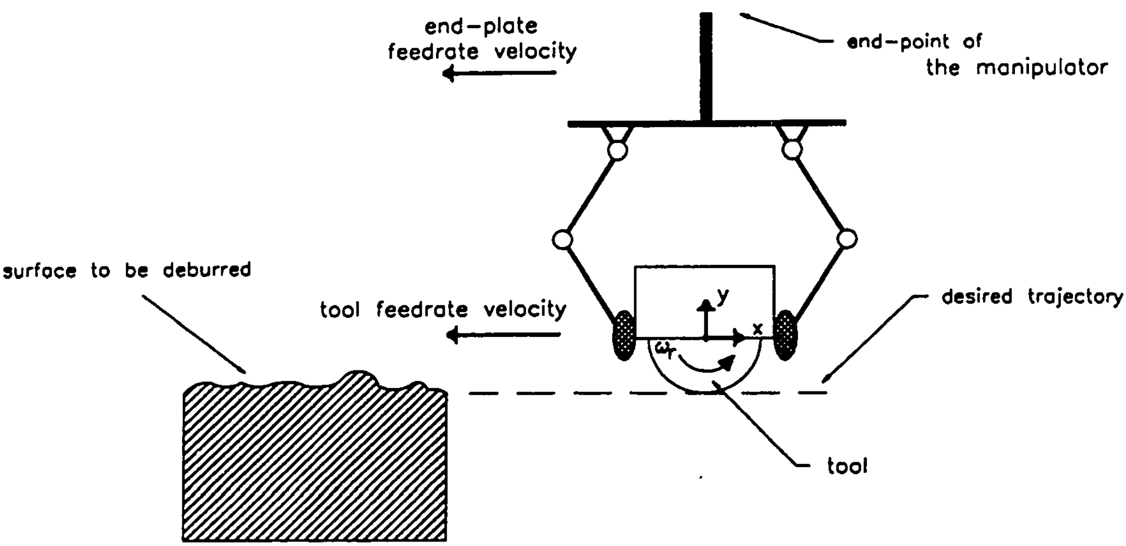


Figure 3.11 - A schematic of a tooling task.

$$f_{d,x} = \bar{f}_{d,x}\sin(\omega_d t) \ ,$$

$$f_{d,y} = \bar{f}_{d,y}\sin(\omega_d t) \ . \tag{3.44}$$

Remark 3.2: The model of the exogenous disturbance given in equation 3.44 assumes that the feedrate velocity of the grasped tool and the end-plate of the manipulator are the same. In the actual implementation, there may be a difference between these two velocities, i.e. (tool feedrate velocity < end-plate feedrate velocity). As a result, the models of the components of the exogenous disturbance wrench acting on the finger end-point reference coordinate frame may be written as:

$$f_{d,x} = \bar{f}_{d,x}\sin(\omega_d - \Delta\omega_d)t \ ,$$

$$f_{d,y} = \bar{f}_{d,y}\sin(\omega_d - \Delta\omega_d)t \ ,$$

where $\Delta\omega_d$ represents the change in the angular velocity of the components of the disturbance wrench due to the difference in the feedrate velocities. However, in this thesis it is assumed that $\frac{\Delta\omega_d}{\omega_d} << 1$ and the models of the components of the exogenous disturbance along the finger end-point reference coordinate frame are as given by equation 3.44.

In general, the above components are solutions to the following linear differential equations defined by:

$$\ddot{f}_{d,x} + k_{d,x} f_{d,x} = 0 \ ,$$

$$\ddot{f}_{d,y} + k_{d,y} f_{d,y} = 0 \ . \tag{3.45}$$

The method for designing robust control of a grasping wrench for the tooling task is based on the direct application of the solution to a general servomechanism problem. For this application, the objectives are to follow the desired step input in the desired grasping wrench regardless of the presence of the unwanted disturbance wrench which arises during the tooling task and the presence of the friction wrenches in each joint.

From previous section, the roots of the characteristic equation representing the constant disturbance wrench are all zero, i.e. $s_{1,x}=s_{1,y}=0$ in equation 3.35. The roots of the characteristic equation of the input exogenous disturbance due to the tooling task along each axis of the finger point, i.e. a *2DOF* finger can be obtained from equation 3.45 to be $s_{2,x} = \pm j\sqrt{k_{d,x}}$ and $s_{2,y} = \pm j\sqrt{k_{d,y}}$ where $j = \sqrt{-1}$.

From equation B.21 and the definition of the *servo-compensator*, we have:

$$\begin{Bmatrix} \dot{\xi}_{1,x} \\ \dot{\xi}_{2,x1} \\ \dot{\xi}_{2,x2} \\ \dot{\xi}_{1,y} \\ \dot{\xi}_{2,y1} \\ \dot{\xi}_{2,y2} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -k_{d,x} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -k_{d,y} & 0 \end{bmatrix} \begin{Bmatrix} \xi_{1,x} \\ \xi_{2,x1} \\ \xi_{2,x2} \\ \xi_{1,y} \\ \xi_{2,y1} \\ \xi_{2,y2} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} e_x \\ e_y \end{Bmatrix} . \tag{3.46}$$

or,

$$\dot{\xi} = \Lambda\xi + \beta e \ .$$

Let the model of a finger with soft finger-tip in contact with the grasped object be given by equation 3.13, or:

$$\dot{T}_{fe} = W^{e}_{act} \ . \tag{3.47}$$

Which can be written in state-space form as:

$$\dot{x} = \mathbf{A}^o x + \mathbf{B}^o u \ , \tag{3.48}$$

where :

$$\mathbf{A}^o = \begin{bmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix} \; ; \; \mathbf{B}^o = \begin{bmatrix} 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \end{bmatrix} .$$

The output vector of the system is given by equation 3.19, or:

$$y = \mathbf{C}\,x = \begin{bmatrix} k_{u,x} & 0 & c_{u,x} & 0 \\ 0 & k_{u,y} & 0 & c_{u,y} \end{bmatrix} x. \tag{3.49}$$

Similar to the section 3.5, the above nominal system satisfies the stablizability and observability conditions. The condition of equation B.17 for the above system is written as:

$$\begin{bmatrix} -\nu & 0 & 1 & 0 & 0 & 0 \\ 0 & -\nu & 0 & 1 & 0 & 0 \\ 0 & 0 & -\nu & 0 & 1 & 0 \\ 0 & 0 & 0 & -\nu & 0 & 1 \\ k_{u,x} & 0 & c_{u,x} & 0 & 0 & 0 \\ 0 & k_{u,y} & 0 & c_{u,y} & 0 & 0 \end{bmatrix} . \tag{3.50}$$

By selecting ν equal to the roots of characteristic equations of the exogenous inputs defined before, namely, $s_{1,x}, s_{1,y}, s_{2,x}, s_{2,y}$, it can be seen that the rank of the matrix in equation 3.50 is always equal to 6, i.e. for a *2DOF* finger. The results of this condition state that the roots of the characteristic equations of the exogenous inputs do not coincide with the transmission zeros of the system ($\mathbf{A}^o$,$\mathbf{B}^o$,$\mathbf{C}$). Therefore, the conditions for the existence of the robust controller given the above models of the exogenous inputs are satisfied.

Based on the procedure which is outlined in *section B.4,* the objective here is to combine the models of servo-compensator defined in equation 3.46 with the nominal model of the system defined above for the case when all the input exogenous inputs are set to zero to obtain the following:

$$\begin{Bmatrix} \dot{x} \\ \dot{\xi} \end{Bmatrix} = \begin{bmatrix} \mathbf{A}^o & \mathbf{0} \\ -\beta\mathbf{C} & \Lambda \end{bmatrix} \begin{Bmatrix} x \\ \xi \end{Bmatrix} + \begin{bmatrix} \mathbf{B}^o \\ \mathbf{0} \end{bmatrix} u \; , \tag{3.51}$$

or :

$$\dot{\bar{z}} = \bar{\mathbf{A}}\bar{z}+\bar{\mathbf{B}}u \ . \tag{3.52}$$

Applying the control law of equation B.26 to the system defined in equation 3.52, the following closed-loop system is obtained:

$$\dot{\bar{z}} = [\bar{\mathbf{A}}+\bar{\mathbf{B}}\hat{\mathbf{K}}]\bar{z} \ , \tag{3.53}$$

where :

$$\hat{\mathbf{K}} = [\mathbf{K}_{sta} \ ; \ \mathbf{K}_{ser}] \ . \tag{3.54}$$

Here, the gain matrices are selected such that they will result in a stable closed-loop system, e.g. pole-placement or optimal control. However, since the nominal system for this case represents a set of *SISO* transfer functions, the stability conditions can be found using similar approach which was outlined in the previous section, for example in equation 3.41.

Figure 3.12 shows the control block diagram architecture of this application of the robust controller along the x-direction of the finger end-point. Figure 3.13 shows an example of a time response of the disturbance wrench along the x-direction. Figure 3.14 shows the step response of the robust controller when the disturbance wrench of figure 3.13 is presented. As it can be seen from the figure, the controller follows the desired step grasping wrench input along the x-direction of the finger end-point while rejecting the unwanted input disturbance wrench, $f_{d,x}$.

The simple example of this section demonstrated how the general solution to the servomechanism problem can be applied for controlling a tooling task with the grasped object. The model of the disturbance wrench, obtained from a skilled human operator, can be analyzed using Fourier transformation. This model can then be included in the servo-compensator. Figure 3.15 shows an example of how all the frequency components

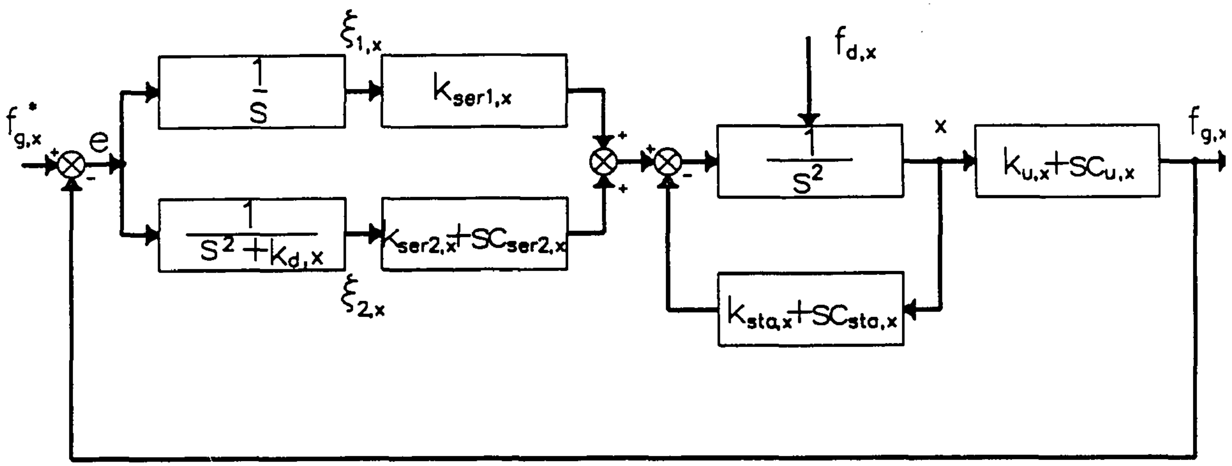


Figure 3.12 – Robust grasping wrench controller for tooling tasks.

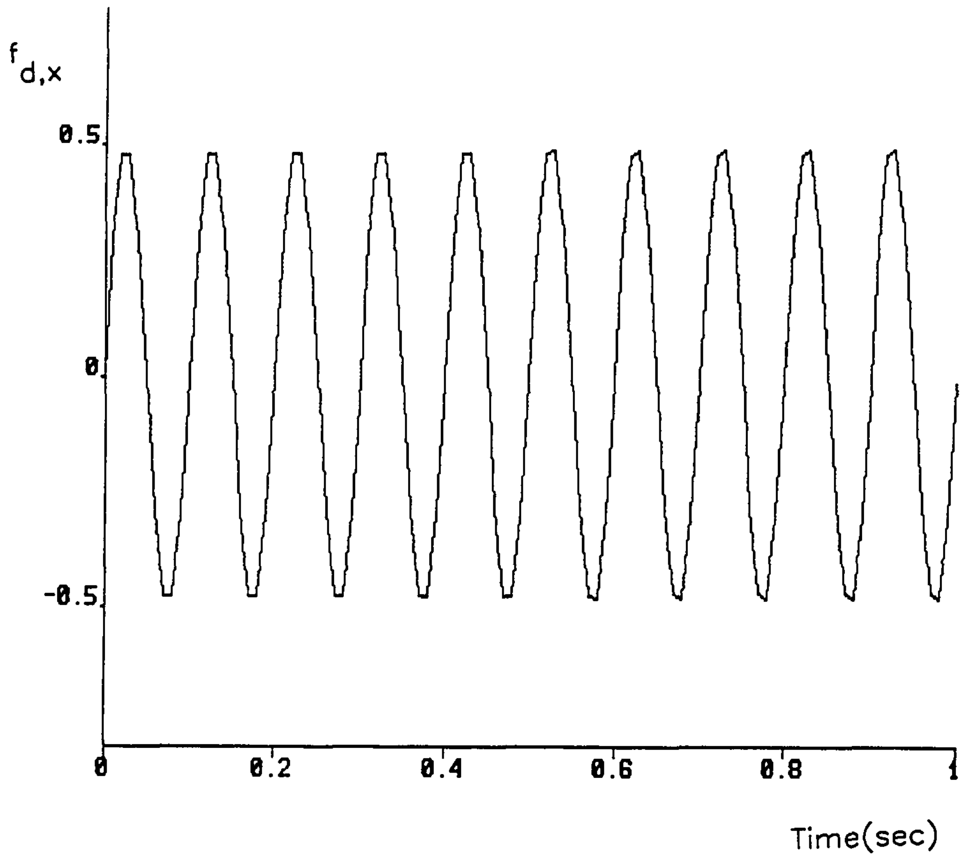


Figure 3.13 – A time response of a component of the disturbance wrench along the x–direction.

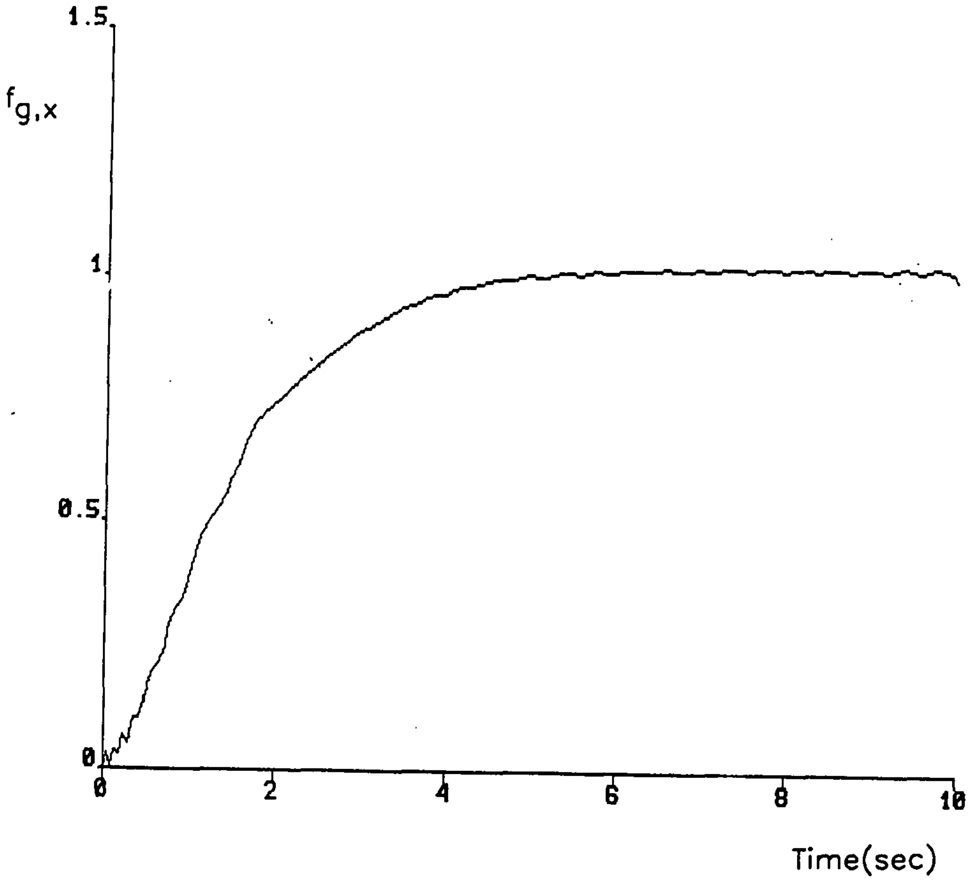


Figure 3.14 – A step response of the robust controller for a tooling task.

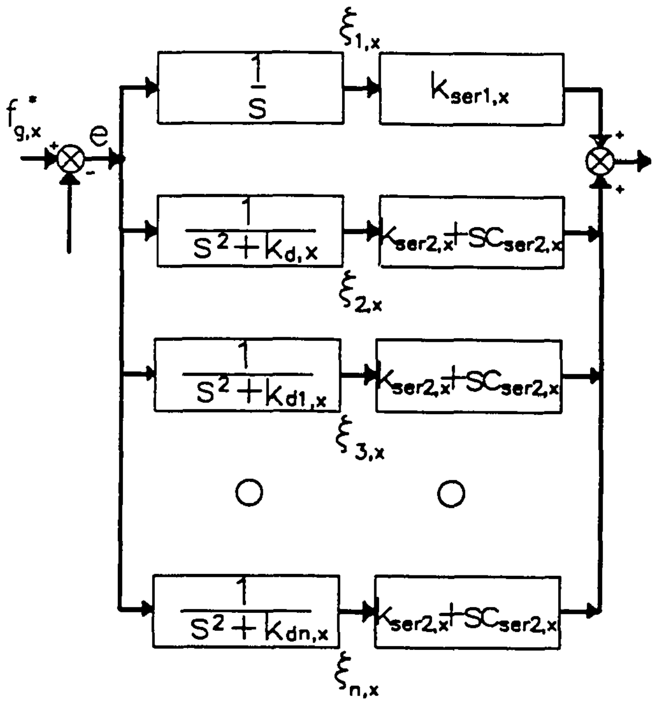


Figure 3.15 – A complete model of the servo–compensator for a tooling task along the x–direction.

of the disturbance wrench along the x-direction of the finger end-point can be constructed in the model of the servo-compensator along the x-direction.

3.8 Summary

An architecture for the independent control of the grasping wrench of each finger was presented. The architecture was obtained based on the input/output relationships of the matched model of a finger and the model of the soft finger-tip. It was shown that the architecture did not have the desired performance. That is, the closed-loop response of the grasping wrench did not have the desired performance when there was a constant disturbance wrench acting on the finger end-point and there existed uncertainties in the model parameters of the finger.

Based on the theory of the servomechanism problem, the definitions of the impedance/admittance blocks of the general architecture were modified such that the architecture is robust. Specifically, the architecture is robust to constant disturbance and to the uncertainties in the actual linear model parameters of the finger. Performance of this controller was demonstrated using a simulated example. The next chapter will demonstrate the actual performance of the above controller using experimental setups.

Furthermore, based on the general theory, a robust grasping wrench controller was proposed for tooling tasks. For these tasks, a model of the exogenous disturbance wrench which arises from the interaction of the tool with the environment was included in the control architecture. The performance of this controller was demonstrated using a simulated example.

CHAPTER IV
Experimental Verification of the Robust Controller

Based on the theory of the servomechanism problem, given a linear model of the system and if the system satisfies certain conditions which are outlined in *section B.4*, then there exists a robust controller for the system. The previous chapter presented the nominal model of the finger with soft finger-tip in contact with an object and showed that the nominal system satisfies all the conditions for the existence of a robust controller. Also, based on this theory, the feedforward admittance/impedance blocks of the general grasping wrench controller are redefined such that the controller is robust to constant exogenous inputs and variations in the nominal system model parameters.

This chapter presents the experimental results on the performance of the robust grasping wrench controller of a finger. In general, the architecture for controlling the grasping wrenches between the fingers of a dexterous mechanical hand and the grasped object has been proposed by controlling each finger independently. This can be done by developing a high level controller, i.e. a grasp planner, in the hand control hierarchy which assigns the desired grasping wrench W_g^*, targeted impedance $\mathbf{Z}_t$ and $\mathbf{K}_{ser}$ for each finger. This *information* is passed to the low level controllers, which are the robust grasping wrench controllers, see Figure 4.1. The high level controller also acts as the coordinator of the fingers during the grasping and further manipulation of the grasped object.

In the Robotics and Automation Laboratory (*RAL*), experimental setups have been built for testing and verifying results related to grasping, manipulation and constrained motion control. This chapter presents the experimental results on the performance of the robust grasping wrench controller using only a 2*DOF* planar finger.

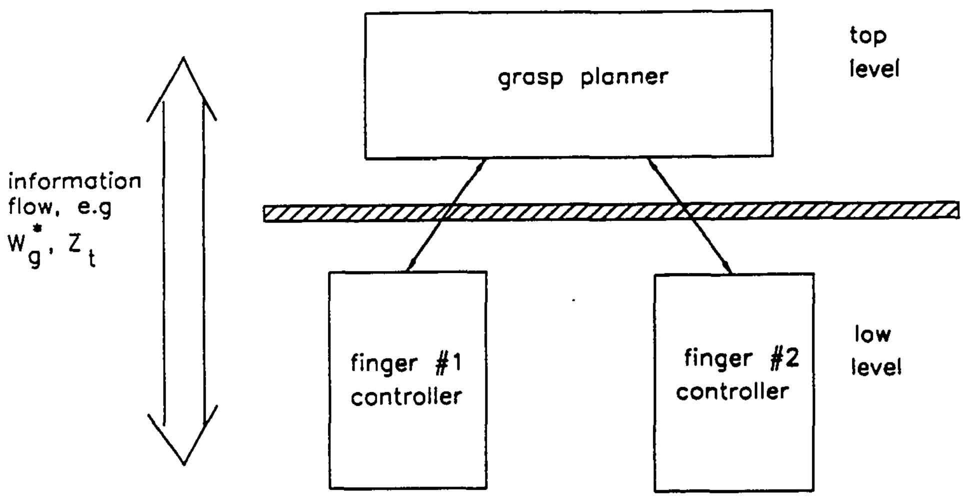


Figure 4.1 – Dexterous mechanical hand control architecture for grasping.

The chapter is organized as follows: section 4.1 describes the hardware configuration of the experimental setup; section 4.2 presents software development for executing the grasping task; section 4.3 presents experimental results of the robust grasping wrench controller and finally section 4.4 discusses the results of this chapter.

4.1 The Experimental Setup

This section presents the hardware configuration which is used to investigate the performance of the robust grasping wrench controller.

In general, the grasping setup consists of two planar two degrees of freedom fingers, i.e. manipulators, Shimoga, Lu and Payandeh[30], see Figures 1.4a and 4.2. Each finger has a five bar mechanism configuration where all the active joints are located at the base of the finger. This configuration has an advantage that the weights of the driving motors are concentrated at the base of the manipulator. The joints are activated through direct drive motors. These motors have an advantage that their output shaft is directly attached to the link of the manipulator without any transmission mechanism, e.g. gears.

The experimental results of this chapter are concerned only with the control of the grasping wrench of a single arm with a soft finger-tip. The soft finger-tip is constructed using a compression spring which is placed in a cylindrical compartment, see Figure 4.3. This configuration only allows the displacement of the spring along its longitudinal direction. A spring with the known spring constant of $3.5\times10^3\ N/m$, SPEC catalog[31], represents the actual soft finger-tip model. The mechanical configuration of Figure 4.3 allows the finger-tip to be adjusted such that the centerline axis of the cylindrical compartment can always be perpendicular to the rigid wall. This can be done by turning the finger-tip setup about the center-pin and by tightening the two bolts which are free to move in the circular grooves.

In order to allow the manipulator to have a reference configuration, limit switches are installed at each joint, see Figure 4.4, so the manipulator can have a reference or

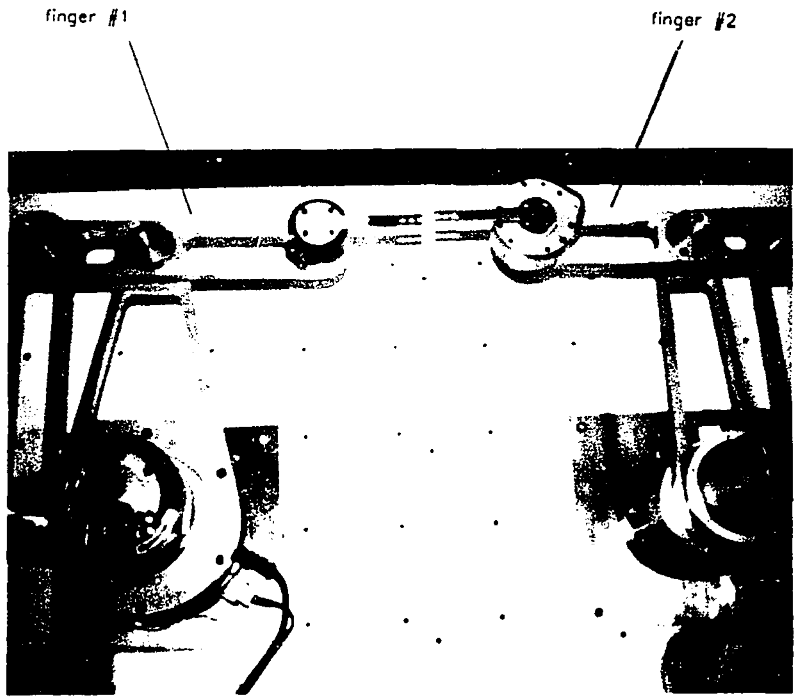
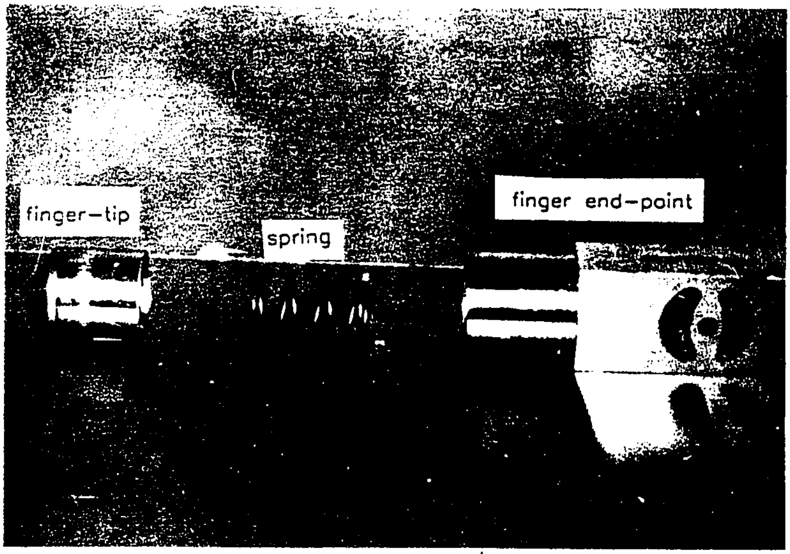
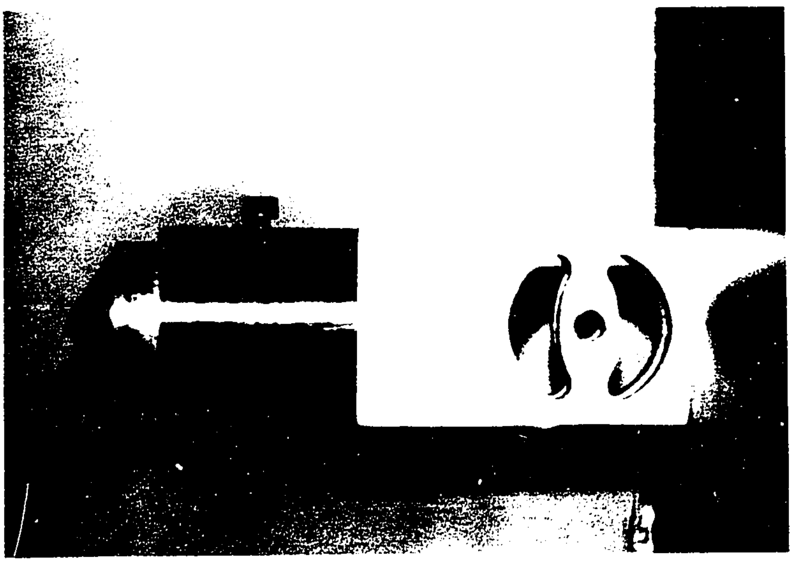


Figure 4.2 – Experimental setup configuration for performing grasping experiments.



a) disassembled



b) assembled

Figure 4.3 – The configuration of the soft finger-tip.

home configuration. This allows the motion of the manipulator to be determined with respect to the fixed reference axes.

4.2 Software Development

The high level software development for investigating the performance of the grasping wrench controller of a finger is divided into two parts: 1) home positioning; and b) grasping wrench controller.

4.2.1 Home Positioning

The home positioning algorithm performs the calibration of the finger. This is done by turning each motor using a velocity controller until a designated limit switch is closed. Then by zeroing the corresponding register for the position values, the clockwise and counterclockwise motion of the each motor can be determined from this reference zero register. The algorithm has a feature that the arm can be in any configuration in its workspace prior to executing the zeroing configuration.

The above zeroing algorithm did not result in an arm configuration which coincided with the kinematic reference or home configurations. In order to determine the difference between the limit switches zeroing configuration and the kinematic home configuration, a calibration procedure was carried out. The procedure was based on locating a number of predefined points in the workspace of the manipulator. Then, by comparing the joint positions obtained from the inverse kinematic solutions and the actual position read-outs from the joint registers, the differences between the zeroing and the kinematic home positions were determined.

4.2.2 Robust Grasping Wrench Controller

The control algorithm for the grasping wrench is based on the results of the *chapter III*. Since the configuration of a finger is planar, i.e. the plane being the horizontal plane,

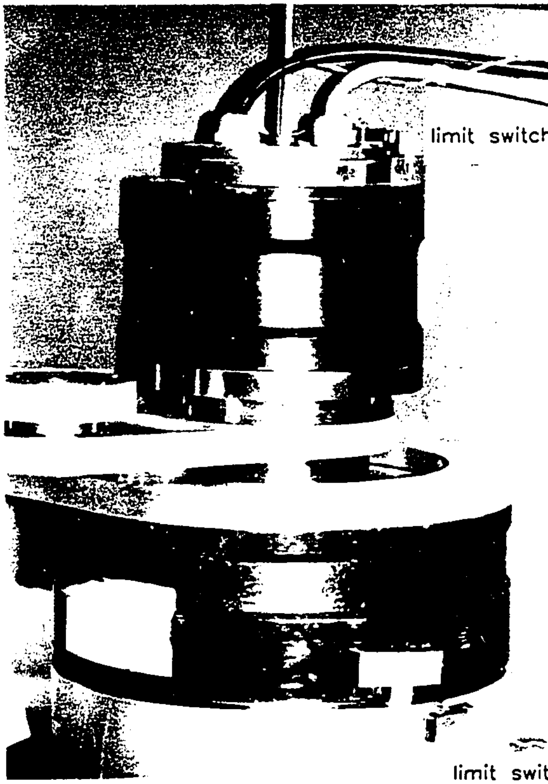


Figure 4.4 – The locations of the limit switches.

the effect of gravity in the dynamic model of the finger is neglected. Also, since the finger-tip is only pushing on the wall, see Figure 4.5, the velocity dependent components in the dynamic equation of the finger are further ignored. As a result, the simplified dynamic model of the finger with soft finger-tip in contact with the obstacle wall is written as: (see also *remark D.2,* and equation 3.11)

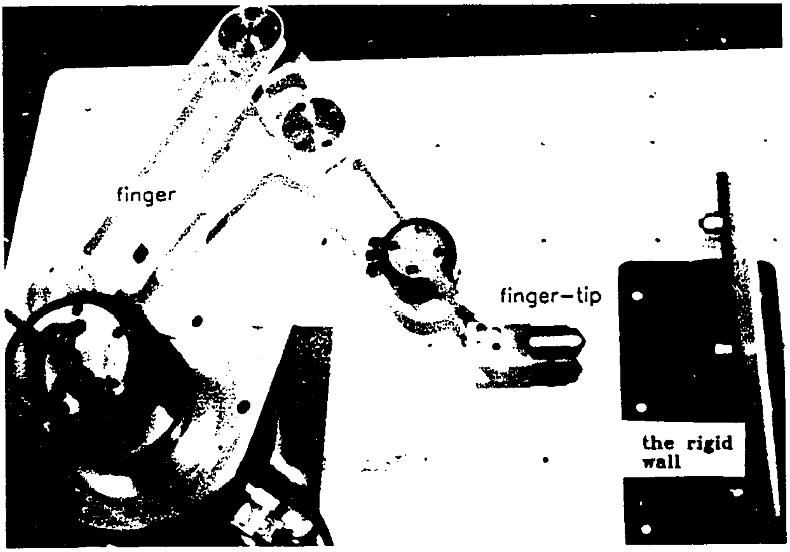
$$\mathbf{M}_{fe}\dot{T}_{fe}+W_g=W_{act} \ , \tag{4.1}$$

where $\mathbf{M}_{fe}$ is the inertia matrix of the finger expressed in the end-point coordinate frame, or $\mathbf{M}_{fe} = \mathbf{J}_\theta^{-t}\mathbf{M}_\theta\mathbf{J}_\theta^{-1}$, see *Appendix D*. The matrix $\mathbf{M}_\theta$ is obtained by calculating the exact mass and moment of inertia parameters of the finger, Shimoga, Lu, Payandeh[32]. The configuration parameters of the finger at the contact location with the rigid wall are used to obtain the Jacobian of a finger $\mathbf{J}_\theta$ and the linear parameter of the dynamic model of the finger, namely $\mathbf{M}_{fe}$. This mass matrix is further modified to include the mass and inertia of the soft finger-tip attachement.

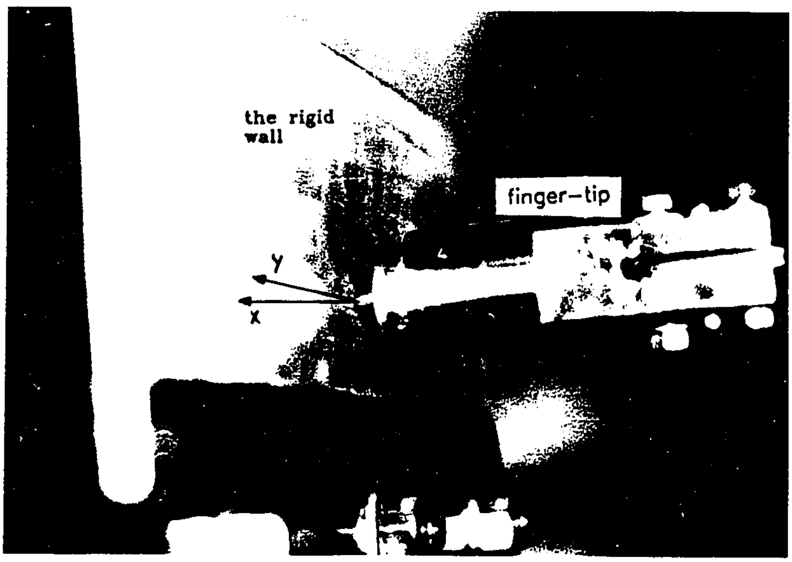
The objective of the experiment is to show the actual performance of the robust grasping wrench controller proposed in the *chapter III*. In this experiment, the finger-tip was brought into contact with the (obstacle) wall where the wall surface is parallel with the y-direction of the finger end-point reference coordinate frame, Figure 4.5b. At the contact location with the wall, the arm is adjusted such that the center-line axis of the soft finger-tip is perpendicular to the wall in the x-direction of the finger end-point. The objective is for the finger-tip to push on the wall in the x-direction with a desired force and to investigate the response of the controller. For these experiments, the actual component of the grasping wrench along the x-direction is calculated using the exact model of the spring and the displacement of the finger end-point , or:

$$f_{g,x}=k_{u,x}\int v_x dt \ . \tag{4.2}$$

Where $\int v_x dt$ is the displacement of the finger end-point from the initial contact with the



a) finger–tip location before contact



b) finger–tip location during contact

Figure 4.5 – The preocess of finger #1 making soft contact with a rigid wall.

wall in the x-direction and $k_{u,x}$=3.5×10^3 N/m. As was explained in *section 2.3*, the component of the grasping wrench $f_{g,x}$ is also the component of the external wrench which is acting on the finger. Also, since the finger only exerting force along the x-direction of the finger end-point, the component of the grasping wrench along the y-direction is equal to zero.

The matching condition of equation 2.9 is written as:

$$\mathbf{Z}_s = \begin{bmatrix} m_{t,x}-1 & 0 \\ 0 & m_{t,y}-1 \end{bmatrix} s^2 + \begin{bmatrix} c_{t,x} & 0 \\ 0 & c_{t,y} \end{bmatrix} s + \begin{bmatrix} k_{t,x}+k_{u,x} & 0 \\ 0 & k_{t,y} \end{bmatrix} . \tag{4.3}$$

As it can be seen from the above matching condition, since the soft finger-tip is modeled as a spring along the x-direction of the finger end-point, the impedance of the soft finger-tip only appear along the x-direction. Figure 4.6 shows the control block diagram of the experimental implementation. This control block diagram shows the closed-loop control of $f_{g,y}$. However, the desired magnitude for the component was set to zero since the finger-tip is not in contact with the rigid wall in the y-direction.

4.3 Performance of the Robust Grasping Wrench Controller

This section presents some results regarding the actual response of the controller of Figure 4.6 along the x-direction of the finger end-point. The following cases show the step responses of the controller to the 1.0 N force as a desired grasping wrench along the x-direction of the finger end-point. The gain parameters are selected such that they result in a stable closed-loop system, i.e. the gain parameters satisfy the inequalities of 3.41.

Case #1:

$m_{t,x}$=1.0 Kg, $c_{t,x}$=100.0 $N/(m/sec)$, $k_{u,x}$=3.5×10^3 N/m, $k_{t,x}$=1 N/m, $k_{ser,x}$=35

In this case, the servo-compensator gain is selected such that the controller has the desired response along the x-direction of the finger end-point as it is shown in Figure 4.7.

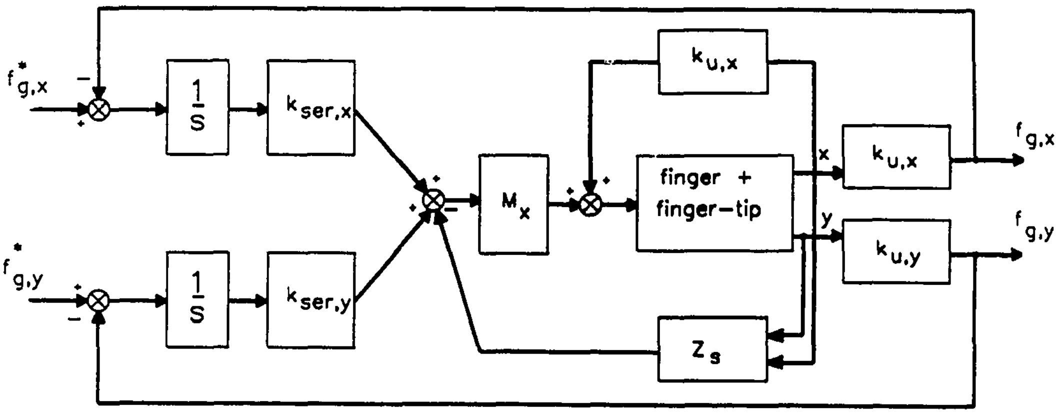


Figure 4.6 – The robust control block diagram of the experimental implementation.

Case # 2 :

The gain parameters of this case is the same as case #1 and the objective of this experiment is to investigate the performance of the controller when the actual model of the finger is different from the one calculated before, e.g. $\mathbf{M}_\theta$. For this case, a block of weight equal to 1.0 *Kg* is added to the last link of the finger. Figure 4.8 shows the step response of the controller. As it can be seen from the Figure, $f_{g,x} \approx 1.0\,N$ after 0.8 *sec*. Although the transient response of the controller has an overshoot but, the response has negligible steady-state error, i.e. the performance of the system is independent from the variation in the model parameters of the system.

Case # 3 :

This case is similar to the previous case but a block of weight equal to 2.0 *Kg* was added to investigate the performance of the controller. As it can be seen in Figure 4.9, the response of the controller is similar to the previous response, except the controller has a bigger overshoot. The above two cases lead to the conclusion that if there are uncertainties in the model of the finger, the actual response of the grasping wrench along the x-direction of the finger end-point follows the desired step in grasping wrench asymptotically. However, the transient response of the controller is affected by this perturbation.

Case # 4:

The objective of the following experiments is to investigate the effect of the servo-compensator and its corresponding gain parameters. First, the feedforward blocks which include the servo-compensator and its gain has been removed. It was noticed that the controller did not responded at all. This was due to the unmodelled friction wrench which is present in each joint of the finger. Next, the parameter of the servo- compensator gain was changed to 5.0 and 10.0 respectively from the gain parameter defined in the case #1. Figure 4.10 and Figure 4.11 shows the response of this controller for a step in

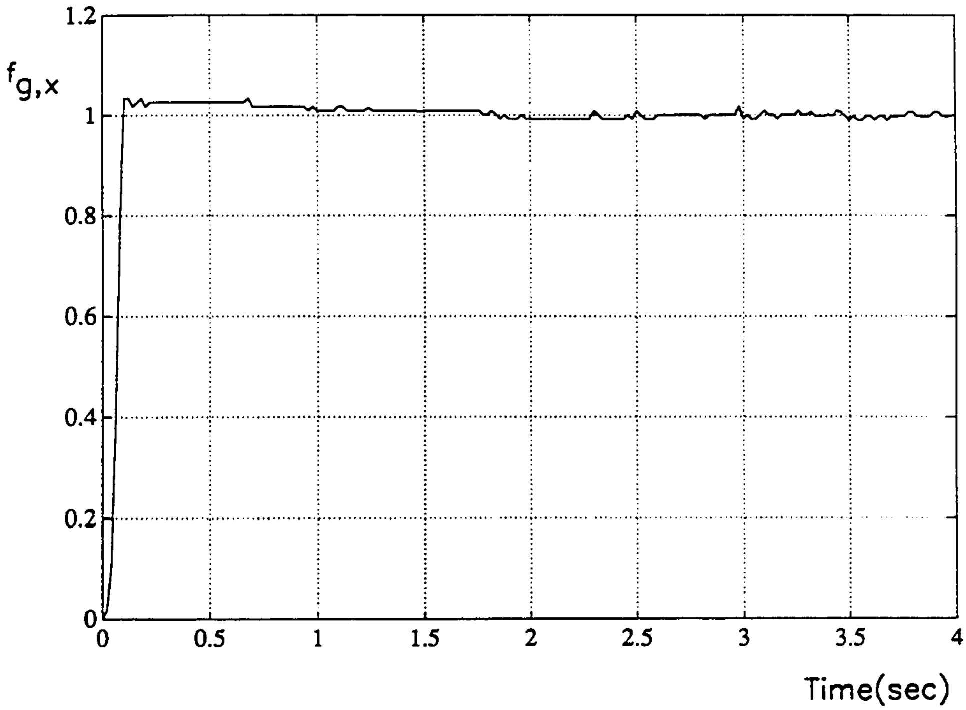


Figure 4.7 – Experimental response of the robust controller for the case #1.

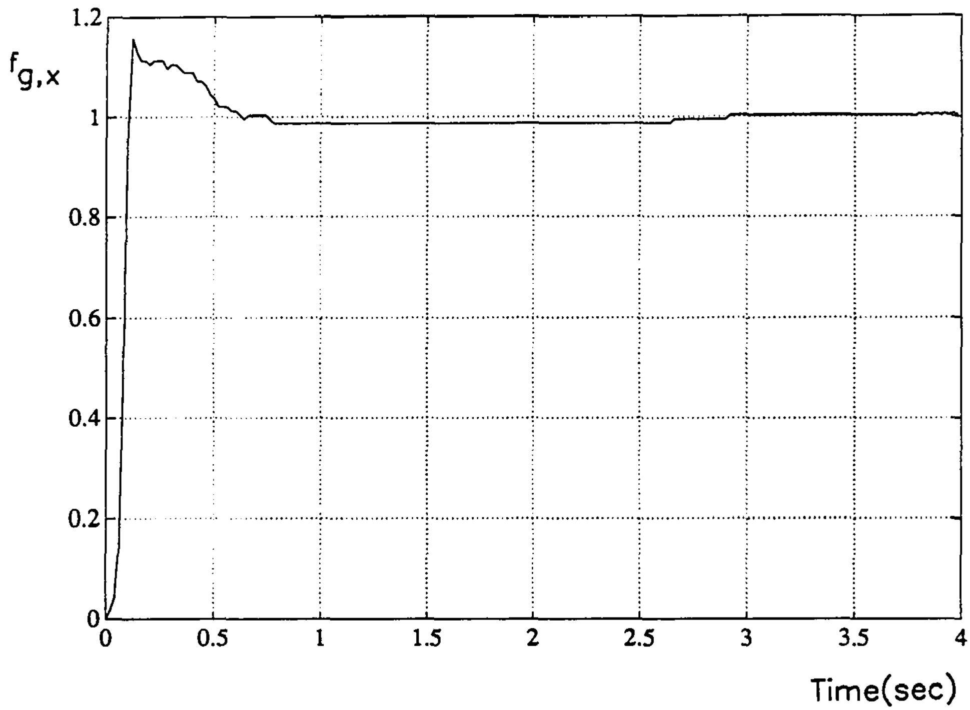


Figure 4.8 – Experimental response of the robust controller for the case #2.

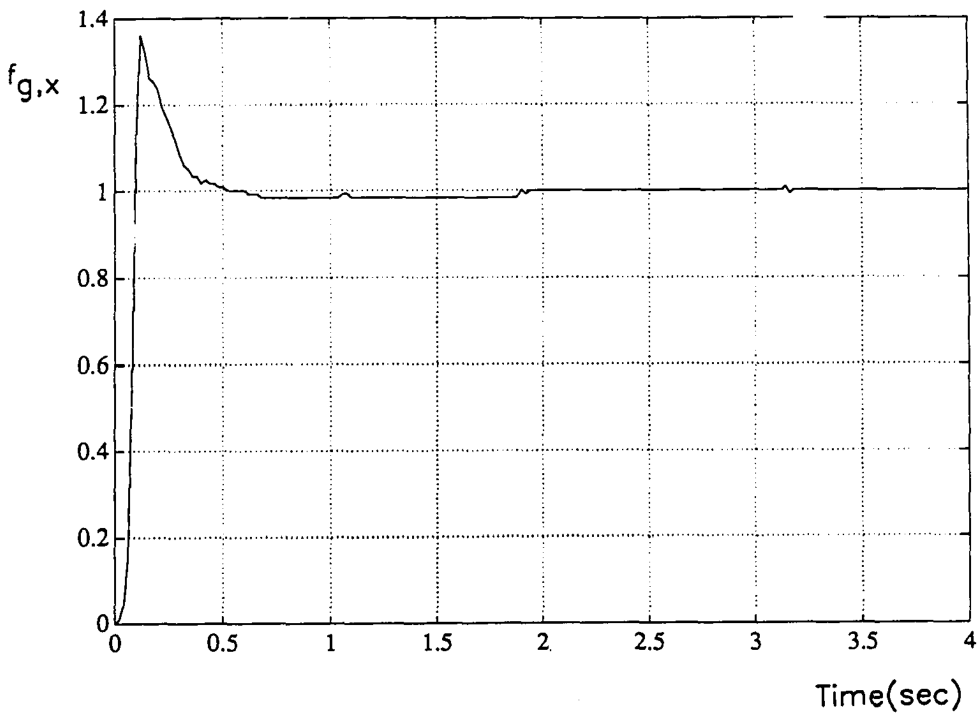


Figure 4.9 – Experimental response of the robust controller for the case #3.

the desired grasping wrench. It can be seen from the figures that the actual response of the grasping wrench approaches the desired response. However the settling time and the overshoot are affected by decreasing the servo-compensator gain.

Case #5

The objective of this experiment is to investigate the performance of the controller when the finger-end point has been pushed away from the steady-state magnitude. As it can be seen from the figure 4.12, e.g. $k_{ser,x}$=20, the controller asymptotically regulates the desired magnitude even in the presence of friction which acts as a constant disturbance wrench in each active joint of the finger.

4.4 Summary

This chapter presented the experimental results of the performance of the robust grasping wrench controller. Specifically, these results were concerned with the performance of the grasping wrench controller implemented on a single *2DOF* planar finger with the soft finger-tip in contact with a rigid wall.

It was shown that the controller performed as well as it was expected from the results of the previous chapter. For example, the controller is robust to variation in the model parameters of the finger and the presence of the disturbance wrench which can arise from the collision of the grasped object with the moving obstacle in the work-cell.

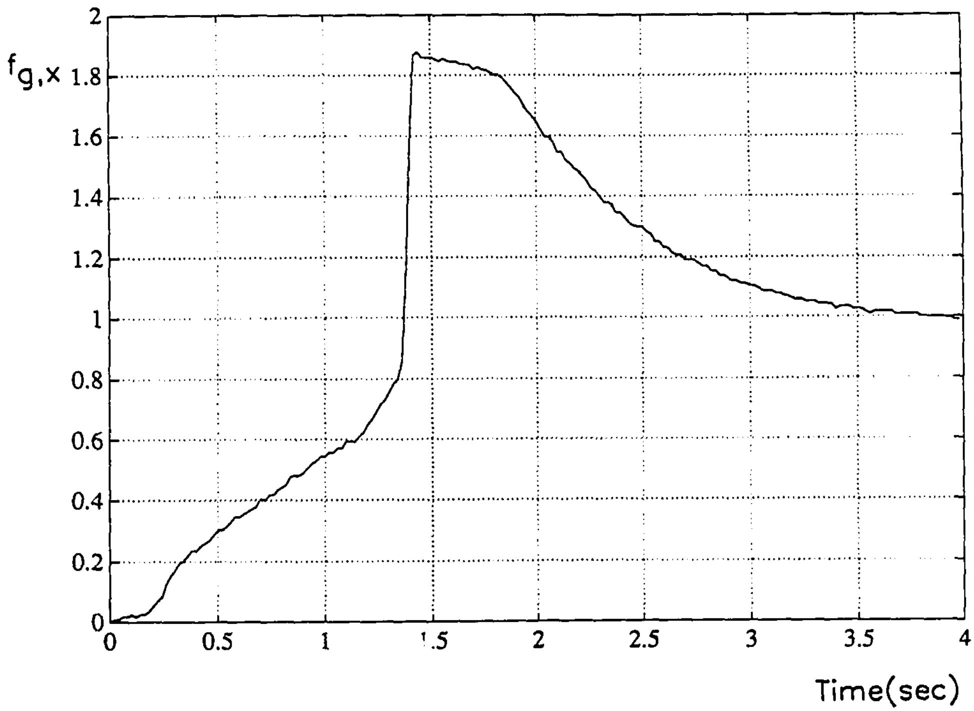


Figure 4.10 – 1st experimental response of the robust controller for the case #4.

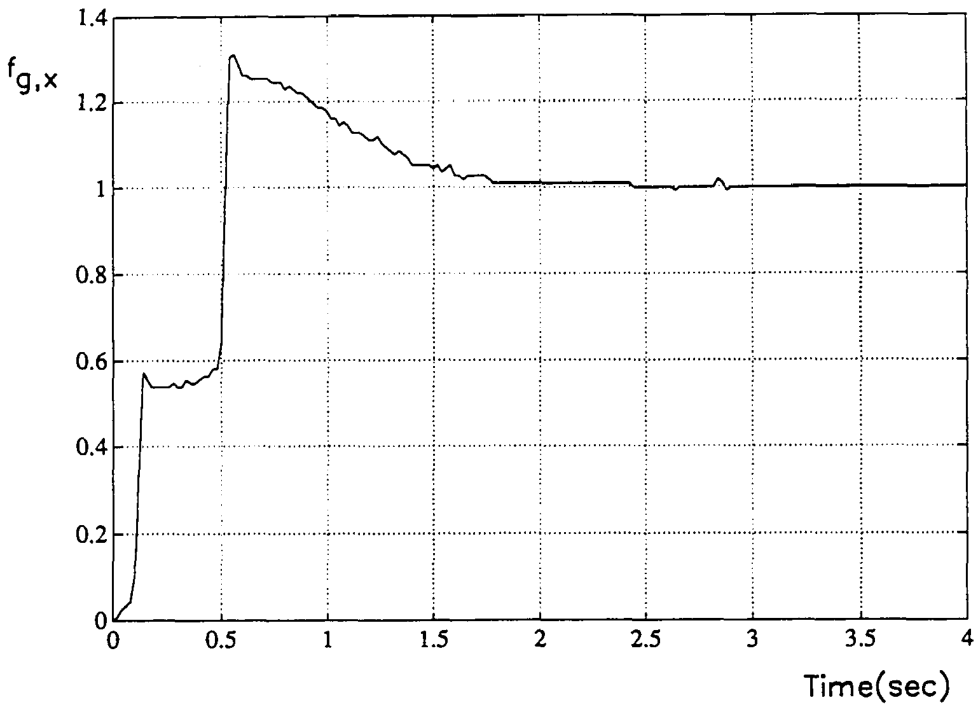


Figure 4.11 – 2nd experimental response of the robust controller for the case #4.

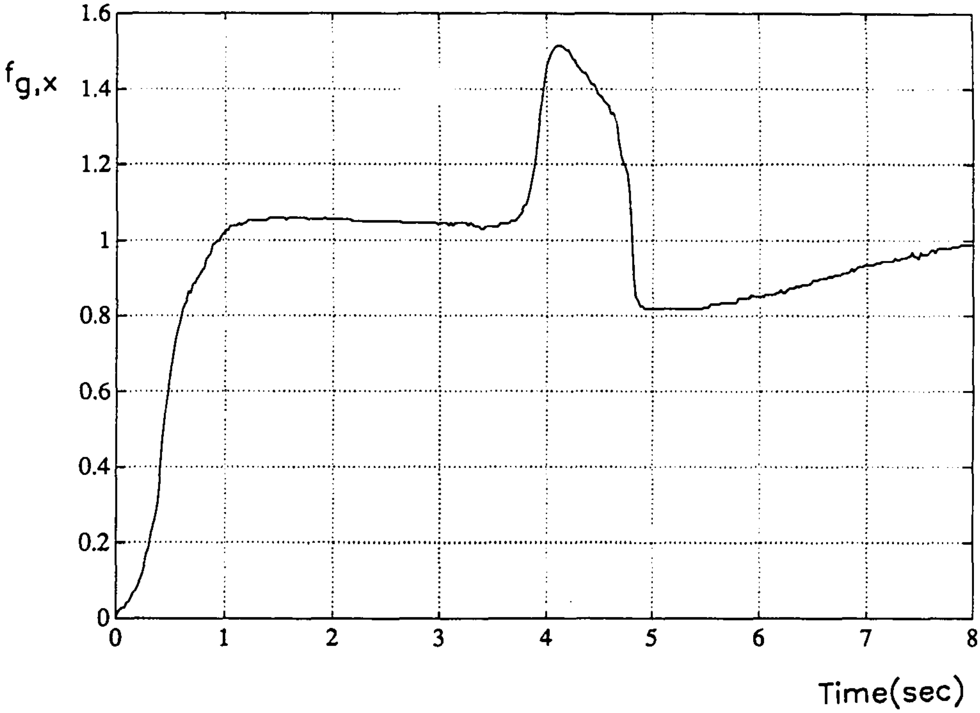


Figure 4.12 – Experimental response of the robust controller for the case #5.

CHAPTER V
Discussion, Conclusion and Future Research

This chapter presents the discussion and conclusion of the results of this thesis while outlining the future research.

5.1 Discussion and Conclusion

Grasping is an area of research which involves modelling, analysis and control of the mechanics of interaction between grasp elements namely, fingers, object and the environment. The work of the thesis is a fundamental investigation into the above areas of research.

Based on the causality principle, it was postulated that the elements of a mechanical system must be connected in a specific manner in order to have a causal model of the system. For example, in grasping, each finger was represented by a mass, spring and damper model, *(proposition 2.2).* In that representation, the spring and damper models were connected in parallel between the mass model of the finger and the palm of the hand. Similarly, the model of the soft finger-tip was presented as a spring and damper model that were connected in parallel between the grasped object and the mass model of the grasping finger, *(proposition 2.3).*

This method of modelling a mechanical system can also be extended to model the interaction of the robot manipulator with a rigid environment where the robot manipulator is modelled as a mass, spring and damper. In order to have a causal interaction between the mass model of the robot manipulator and the rigid environment, spring and damper can be introduced between the two mass models. The spring and damper which are introduced at the end-point of the manipulator can be a model of the visco-elastic material attached to the end-point.

The desired linear model of a finger was referred to as the targeted impedance. The parameters of this model were selected such that the response of the grasped object to the external wrenches satisfied a given task requirement. Throughout the thesis it was assumed that these model parameters of each grasping finger were given.

In general, the actual linear impedance of a grasping finger is different from the impedance model specified by the targeted impedance. However, by designing linear feedback control laws, it is possible to match, i.e. replace, the actual impedance with the desired one, i.e. concept of impedance matching, *(section 2.2)*. Based on the assumption that the exact model parameters of the finger were given and there was no disturbance wrench acting on the finger, a method for implementing the targeted impedance of the finger was developed. The method was based on the linear dynamic decoupling control law. Implementation of this method resulted in the actual impedance of the finger to be replaced with the targeted impedance, see equations 2.10 and 2.11.

When fingers of a dexterous mechanical hand are grasping an object, they must exert wrenches on the grasped object equal to the desired grasping wrenches and further maintain these wrenches during a task. In general, these grasping wrenches are given by a top-level controller named the grasp planner. Using the targeted impedance model of the finger and a model of the soft finger-tip, the architecture for the independent control of the grasping wrenches between the fingers and the grasped object was developed. In an ideal case, i.e. when the actual impedance parameters of the finger were exactly known, and when no disturbance was acting on the finger, it was shown that the open-loop architecture can result in the actual grasping wrench to be identically equal to the desired one, (equation 3.5). However, in practical applications there always exist some uncertainties in the model parameters of the finger and also the presence of disturbance wrenches which can act on the finger. For example, it was shown that when a disturbance acts on the finger, the open-loop response of the grasping wrench was not the same as the desired one even when the exact model parameters of the finger were assumed to be known, (equation 3.7).

It was also shown that the closed-loop response of the grasping wrench controller did not have the desired performance. For example when a disturbance wrench was acting in the x-direction of the finger end-point, the closed-loop response of the grasping wrench was not the same as the desired one even when the exact model of the finger-tip was assumed to be known, (equation 3.10). In summary, the general architecture for controlling the grasping wrench based on the impedance matching concept did not result in a practical controller.

In order for the general architecture for controlling the grasping wrench to have a desired performance in a practical applications, based on the theory of the servomechanism problem, the definitions of feedforward control blocks of the architecture were modified in order to exhibit robustness. First, in order for a robust controller to exist, the nominal model of the system including the finger and the finger-tip, must satisfy some conditions, *(section B.4)*. It was shown that the nominal system satisfies all the conditions for the existence of the robust controller, and the feedforward impedance/admittance blocks of the general architecture were redefined such that the architecture exhibited robustness, *(section 3.6)*.

The resulting controller is robust to model parameter uncertainties and the presence of constant disturbance wrenches. In general, the gain parameters of the robust controller can be determined based on the standard methods such as pole-placement or optimization approaches. However, in the present architecture, the stabilizing gains are defined based on the matched impedance of the finger and the impedance model of the soft finger-tip, (equation 3.38). The servo-compensator gains, are selected such that the closed-loop system is stable and has the desired transient response,(equation 3.37, Figure 4.7).

A direct application of the general theory to tooling tasks was presented and demonstrated using an example. In this application, the model of the disturbance wrench which arises from the interaction of the tool with the work-piece was included in the model of the servo-compensator. As a results, two types of exogenous disturbances namely, a constant and a time varying (sinusoidal) can be rejected by the controller.

In order to experimentally verify the performance of the robust grasping wrench controller of a finger, an experimental setup was built in the Robotic and Automation Laboratory. A *2DOF* planar manipulator was used as a finger of a dexterous mechanical hand. A model of the soft finger-tip was constructed using a linear spring with a known stiffness property. In general, a finger-tip can be a rubber type material which covers the tip of the last link of the finger. The exact spring and/or damping properties of the finger-tip material can be determined by experiments or through finite-element analysis.

The response of the controller was tested for a number of cases. It was found that the controller is robust to the variations in the finger dynamic model and the presence of disturbance wrenches. For example, first a set of gain parameters were selected such that they result in a desired response of the controller. Then by changing the mass of the finger and introducing a disturbance, the responses of the controller were recorded. In all of these cases it has been found that the controller had a fast response and asymptotically regulated the error between the actual and the desired grasping wrench to zero.

The response of the controller has not been tested in a multi-fingered dexterous mechanical hand grasping an object. Nevertheless, the experiment with only one finger proves the theoretical basis of the controller and verifies its practical implementation.

5.2 Future Research

Outlined below are some issues for future research.

- **Experimental Grasping-** Chapter IV presented the preliminary experimental results of the robust grasping wrench controller using only a single finger. The objective is to extend the results for controlling the grasping wrench using two or more fingers grasping an object and then performing a tooling task.

- **Models of Soft Finger-tip-** In this thesis, a model of the soft finger-tip was presented as a known linear impedance relationship. This model is a simplified relationship which assumes that a soft finger-tip has a linear visco-elastic properties and the geometrical curvature of the contact surface has no effects in the impedance representation. The objective is to model, e.g. using finite-element approach, various types of soft finger-tips by considering different material and geometrical properties. These models can be stored in a look-up table which can be used by the grasping wrench controllers.

- **Contact Configuration-** The analysis of the thesis was confined to precision grasps, see *definition A.1*, where only finger-tips of a dexterous mechanical hand make contact with the object. The objective is to extend the model to the case where all of the contact areas of fingers and the palm of a dexterous mechanical hand can make contacts with the grasped object.

- **Coarse Manipulation-** Coarse manipulation is defined as the relocation of the grasped object with respect to palm where it is required for the fingers to periodically lose and make contacts with the grasped object, see *definition 1.4.2*. The objective is to investigate the theory of screws and the definition of the ruled surfaces to devise a method for determining the positions and orientations of each finger on the grasped object such that through a sequence of losing and making contacts, the grasped object is relocated within the hand.

• **New Designs of a Dexterous Mechanical Hands -** As it was mentioned in *section 1.2*, most of the current mechanical hand configuration consists of tendon driven mechanisms which are located outside the hand. The objective is to design a mechanical hand based on micro-mechanisms configuration using for example, cams, linkages and gears where the actuating mechanisms can fit into the palm of the hand.

• **Pregrasp Configuration -** The decisions on where the fingers should make contact with the object and how the object should be held between fingers, see *definition 1.2*, is an area of research where knowledge-based approaches have proven to be successful, Payandeh and Goldenberg[8], Nguyen, Payandeh, Poole and Sollbach[33]. The objective is to extend the current results to develop a general grasp planner.

• **Path Planning-** The process for determining suitable paths between points in a work-cell of a arm/hand system is an area of research where the objective is to find a collision free path of a redundant system such that neither the grasped object nor the body of the arm/hand system collide with the fixed obstacles.

• **Hybrid Control of General *6DOF* Manipulator-** The objective of this research is to extend the robust grasping wrench controller results to the hybrid control of the manipulator. In this case, a known visco-elastic material is introduced between the manipulator and the rigid environment. The objectives are to control the force exterted on the environment while controlling the position in the orthogonal directions to the force.

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APPENDIX A
Preliminary Definitions and Assumptions

This appendix presents preliminary definitions and assumptions regarding a model of the finger-tip, definitions of finger/object coordinate frames and a number of transformation matrices.

The appendix is organized as follows: section A.1 presents models of finger-tip of a dexterous mechanical hand, objects and the types of contact that a finger-tip can make with an object; section A.2 defines the reference coordinate frames of the finger/object and presents a number of transformation matrices between these coordinate frames and finally section A.3 summarizes the results.

A.1 Models of Finger-tip, Object and Types of Contact Configurations

This section defines models of finger-tip and objects which are to be grasped. It also presents types of contact configurations which can be established between a finger-tip and an object.

Definition A.1: *A precision grasp* is defined as a grasp configuration where only finger-tips of a dexterous mechanical hand make contact with the object.

Assumption A.1: The grasp configuration of a dexterous mechanical hand is a precision grasp.

Assumption A.2: A finger-tip has soft material properties. These material properties can be represented by a visco-elastic model with spring constant $\mathbf{K}_u$ and damper coefficient $\mathbf{C}_u$.

Assumption A.3: Most objects which are to be grasped and manipulated by a dexterous mechanical hand are *rigid* bodies. Almost all components of an assembly task which are presented to a work-cell are rigid objects, e.g. a peg, and also all the tools which are used in tooling tasks have a rigid handling areas. Also, the environment of the work-cell consists of rigid fixtures.

Based on *assumptions A.2 and A.3,* there are three types of contact that can exist between a finger-tip of a dexterous mechanical hand and an object. These are:

a) vertex contact

b) edge contact

c) plane contact

The following are the definitions of the three types of contacts:

Definition A.2: *Vertex contact* between a soft finger-tip and the object is formed when a finger-tip makes contact with the vertex of an object, Figure A.1a.

Definition A.3 : *Edge contact* between a soft finger-tip and the object is formed when a finger-tip makes contact with the edge of an object, Figure A.1b.

Definition A.4: *Plane contact* between a soft finger-tip and the object is formed when a finger-tip makes contact with the surface of an object, Figure A.1c.

A.2 Finger/Object Coordinate Frames and Transformation Matrices

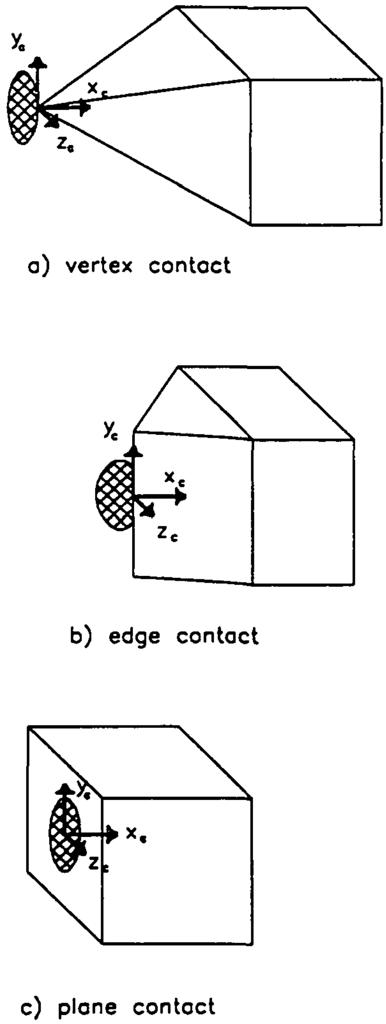


Figure A.1 – Three types of contact between a finger-tip and an object.

This section defines the reference coordinate frames of the finger/object and presents a number of transformation matrices between these coordinate frames.

Definition A.5: A *contact area* is defined as the area common to contacting bodies when two objects are brought into contact, for example, a finger-tip and an object.

Definition A.6: A *port of interaction* is defined as a point in the contact area.

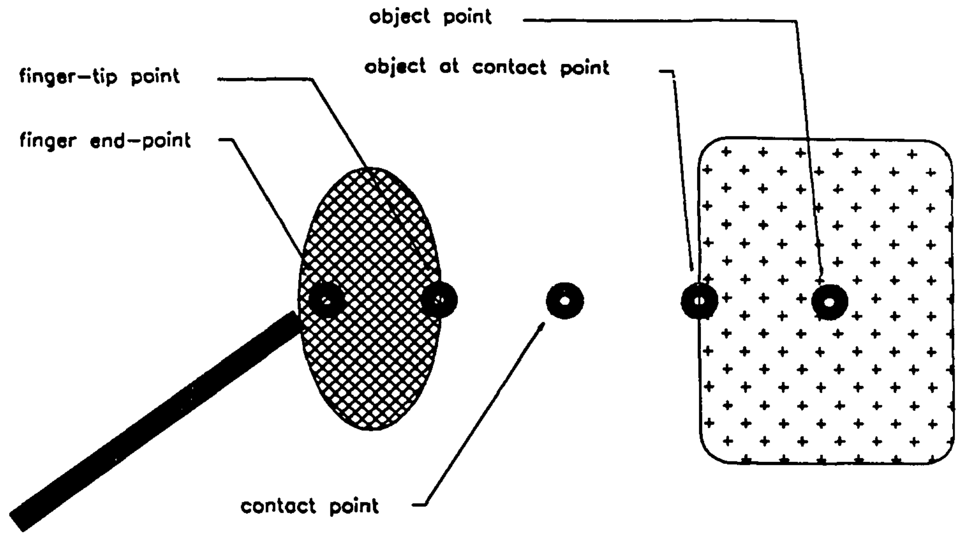
Definition A.7: A *contact reference coordinate frame* is defined as the coordinate frame located at the port of interaction, Figure A.2.

Definition A.8: A *finger-tip reference coordinate frame* is defined as the coordinate frame located on the finger-tip at the contact area, Figure A.2.

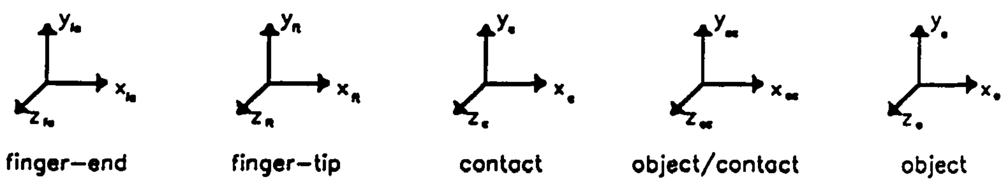
Definition A.9: An *object-contact reference coordinate frame* is defined as the coordinate frame located on the object at the contact area, Figure A.2.

Expressing a wrench, see *definition B.6*, W_{tip} or twist, see *definition B.5*, T_{tip}, of a finger-tip in the contact reference coordinate frame, only some coordinates have nonzero values depending on the type of contact configuration. Similarly, only some coordinates of the wrench W_{oc} and the twist T_{oc} of the object point at the contact area have nonzero values when they are expressed in the same coordinate frame. Figure A.3 shows the nonzero components of wrenches in the contact reference coordinate for the three types of contact configurations.

Definition A.10: $\mathbf{H}_w$ defines the transformation of a wrench expressed in the finger-tip or in the object-contact reference coordinate frames to the wrench expressed in the



a) point locations of the finger/object system



b) finger/object system reference coordinate frames

Figure A.2 – Various coordinate frames of the finger/object system.

contact reference coordinate frame, or: Mason and Salisbury[18], Goldenberg[34]

$$\mathbf{H}_w : \mathbf{R}^{nw} \rightarrow \mathbf{R}^{mw} \ , \tag{A.1}$$

where *nw* is the dimension of the wrench-space of the finger-tip or object-contact and *mw* is the dimension of the wrench-space of the contact.

Remark A.1: The rows of the matrix $\mathbf{H}_w$ are obtained using the bases of the contact reference coordinate frame wrench-space, i.e. see Figure A.3.

Definition A.11: *Constraining-wrenches* are defined as the wrenches expressed in the contact reference coordinate frame.

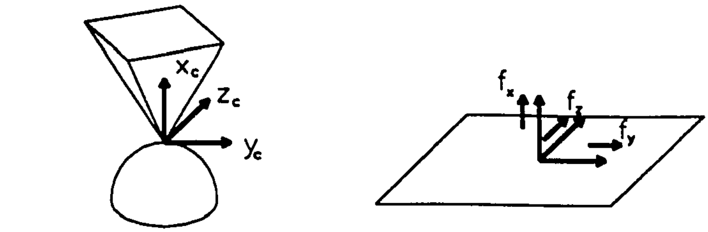
The transformation $\mathbf{H}_w$ for three types of contact configurations are written as:

$$(vertex\ contact) \quad \mathbf{H}_w = \begin{bmatrix} 0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,1 \end{bmatrix} , \tag{A.2a}$$

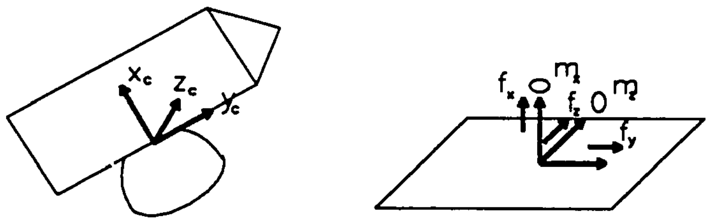
$$(edge\ contact) \quad \mathbf{H}_w = \begin{bmatrix} 1\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,1 \end{bmatrix} , \tag{A.2b}$$

$$(plane\ contact) \quad \mathbf{H}_w = \begin{bmatrix} 1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,1 \end{bmatrix} . \tag{A.2c}$$

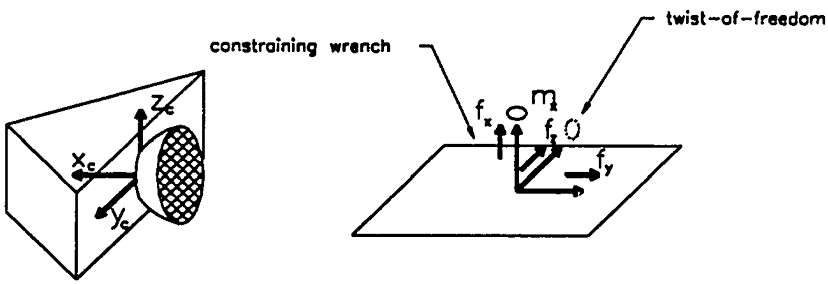
Remark A.2: In the above definition, finger-tip, contact and object-contact reference coordinate frames are always coincide. When a finger-tip or an object-contact reference coordinate frames have differences in orientation with respect to the contact reference coordinate frame, a similar transformation is used which only includes the orientation



a) vertex contact



b) edge contact



c) plane contact

Figure A.3 – The nonzero components of forces and moments expressed in the contact reference coordinate frame.

differences between the coordinate frames, or:

$$\mathbf{H}_w^r : \mathbf{R}^{nw} \rightarrow \mathbf{R}^{mw} \ .$$

Definition A.12: Each *constraining-wrench* results in a twist of the finger-tip or of an object which is defined as a *twist-of-constraint.*

Definition A.13: $\mathbf{H}_t$ defines a transformation of the twist-of-constraints to the twist expressed in the finger-tip or object-contact reference coordinate frames, or:

$$\mathbf{H}_t : \mathbf{R}^{mt} \rightarrow \mathbf{R}^{nt} \ , \tag{A.3}$$

where mt is the dimension of the twist-of-constraint space and nt is the dimension of the twist-space of the finger-tip or object-contact reference coordinate frames.

Remark A.3: The columns of $\mathbf{H}_t$ are obtained using the bases of twist-of-constraint expressed in the contact reference coordinate frame.

The transformation matrix $\mathbf{H}_t$ for the three contact configurations are written as:

$$(\textit{vertex contact}) \quad \mathbf{H}_t = \begin{bmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 1&0&0 \\ 0&1&0 \\ 0&0&1 \end{bmatrix} , \tag{A.4a}$$

$$(\textit{edge contact}) \quad \mathbf{H}_t = \begin{bmatrix} 1&0&0&0&0 \\ 0&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix} , \tag{A.4b}$$

$$(\textit{plane contact})\ \ \mathbf{H}_t = \begin{bmatrix} 1\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,1 \end{bmatrix} \ . \qquad \text{(A.4c)}$$

Theorem A.1: The following relationship exists between $\mathbf{H}_w$ and $\mathbf{H}_t$:

$$\mathbf{H}_t = \mathbf{H}_w^T \ .$$

Proof: Based on the principle of virtual work, see *section B.3,* the reciprocal product between a wrench and a twist, or virtual displacement, expressed in two coordinate frames, i.e. contact and finger-tip coordinate frames, is equal to zero or:

$$\delta(work) = W_c^T T_c = W_{tip}^T T_{tip} = 0 \ .$$

From equation A.1 we have $W_c = \mathbf{H}_w W_{tip}$, therefore, the above equation is then written as:

$$T_{tip} = \mathbf{H}_w^T \, T_c \ .$$

From equation A.3 we have $T_{tip} = \mathbf{H}_t T_c$, then, we can conclude that $\mathbf{H}_t = \mathbf{H}_w^T$. □

Remark A.4: $\mathbf{H}_t$ is the right generalized inverse of $\mathbf{H}_w$ or, $\mathbf{H}_t = \mathbf{H}_w^T(\mathbf{H}_w\mathbf{H}_w^T)^{-1}$, Strang[35]. $\mathbf{H}_w$ is the left generalized inverse of $\mathbf{H}_t$ or, $\mathbf{H}_w = (\mathbf{H}_t^T\mathbf{H}_t)^{-1}\mathbf{H}_t^T$.

Remark A.5: For a given grasp configuration, see *assumption A.1,* the matrices $\mathbf{H}_w$ and $\mathbf{H}_t$ are concatenated to represent the contact configurations of all grasping fingers as $\mathbf{H}_w^c$ and $\mathbf{H}_t^c$, or:

$$\mathbf{H}_w^c = \begin{bmatrix} \mathbf{H}_{w1} & . & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & . & . & . & . \\ . & . & \mathbf{H}_{wi} & . & . \\ . & . & . & . & . \\ . & . & . & . & \mathbf{H}_{wp} \end{bmatrix} , \ \mathbf{H}_t^c = \begin{bmatrix} \mathbf{H}_{t1} & . & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & . & . & . & . \\ . & . & \mathbf{H}_{ti} & . & . \\ . & . & . & . & . \\ . & . & . & . & \mathbf{H}_{tp} \end{bmatrix} .$$

Where $\mathbf{H}_{wi}$ and $\mathbf{H}_{ti}$ are models of contact configurations of the *ith* finger-tip.

Definition A.14: *Grasp Matrix* G defines a transformation of wrenches expressed in the object-contact reference coordinate frames to the wrench expressed in the object reference coordinate frame, or: Mason and Salisbury[18], Goldenberg[34]

$$\mathbf{G} : \mathbf{R}^{nwt} \rightarrow \mathbf{R}^{6} \quad , \tag{A.5}$$

where *nwt* is the dimension of the wrench-space of the object expressed in the object-contact refe-?nce coordinate frames.

Remark A.6: The columns of **G** are obtained using the bases of the wrench-space of the object at the contact areas expressed in the object reference coordinate frame.

Definition A.15: *Grasp Jacobian* **J** defines a transformation of twists expressed in the object-contact reference coordinate frames to the twist expressed in the object reference coordinate frame, or:

$$\mathbf{J} : \mathbf{R}^{ntt} \rightarrow \mathbf{R}^{6} \quad , \tag{A.6}$$

where *ntt* is the dimension of the twist-space of the object at the object-contact reference coordinate frames.

Remark A.7: The columns of **J** are obtained using the bases of the twist-space of the object at the contact areas expressed in the object reference coordinate frame.

Theorem A.2: Given a grasp configuration, i.e. given the location of the object-contact reference coordinate frames, the following relationship exists:

$$\mathbf{G} = \mathbf{J}^{-T} \quad . \tag{A.7}$$

Proof: From the principle of virtual work, the following relationship is written between two coordinate frames of the object:

$$\delta(work) = \overline{W}_{oc}^T \overline{T}_{oc} = W_o^T T_o = 0 \ , \qquad \text{(A.8)}$$

where $\overline{W}_{oc}$ and $\overline{T}_{oc}$ are the combined vector of twists and wrenches expressed in the object-contact reference coordinate frames. From *definition A.15* the following relationship is obtained:

$$T_o = \mathbf{J}\, \overline{T}_{oc} \ . \qquad \text{(A.9)}$$

By substituting equation A.9 into the equation A.8 we have:

$$\overline{W}_{oc}^T \overline{T}_{oc} = W_o^T \mathbf{J}\, \overline{T}_{oc} \ ,$$

or :

$$W_o = \mathbf{J}^{-T}\, \overline{W}_{oc} \ . \qquad \text{(A.10)}$$

From *definition A.14,* the following relationship is defined:

$$W_o = \mathbf{G} \overline{W}_{oc} \ . \qquad \text{(A.11)}$$

Comparing equation A.10 and equation A.11 the following result is concluded :

$$\mathbf{G} = \mathbf{J}^{-T} \ . \qquad \square$$

Definition A.16: $\mathbf{T}_w^c$ and $\mathbf{T}_t^c$ represent the concatenated transformation of the twist and wrench from the finger-tips to the finger end-points reference coordinate frames, or:

$$\mathbf{T}_w^c : \mathbf{R}^{nwtt} \rightarrow \mathbf{R}^{nwtt} \ ,$$

$$\mathbf{T}_t^c : \mathbf{R}^{ntt} \rightarrow \mathbf{R}^{ntt} \ ,$$

where *nwtt* is the dimension of the wrench-space and *ntt* is the dimension of the twist-space.

Definition A.17: A *finger Jacobian* is defined as a transformation of twist from the joint reference coordinate frame of a finger to the finger end-point reference coordinate frame, or:

$$\mathbf{J}_\theta : \mathbf{R}^\theta \rightarrow \mathbf{R}^{nt} \ , \tag{A.12}$$

where θ is the dimension of the twist-space of a finger and *nt* is the dimension of the twist-space of the finger end-point.

Remark A.8: The columns of $\mathbf{J}_\theta$ are obtained using the bases of the twist-space of the finger expressed with respect to the finger end-point reference coordinate frame.

Remark A.9: The virtual work in two reference coordinate frames of a finger can be expressed as:

$$\delta(work) = W_\theta^T T_\theta = W_{fe}^T T_{fe} = 0 \ , \tag{A.13}$$

where W_θ and T_θ are the vectors of joint torques and velocities respectively. From equation A.12 we have :

$$T_{fe} = \mathbf{J}_\theta T_\theta \ .$$

Substituting this relationship into equation A.13 we obtain:

$$W_\theta = \mathbf{J}_\theta^T \, W_{fe} \ . \tag{A.14}$$

Remark A.10: For a given grasp configuration, the matrix $\mathbf{J}_\theta$ is represented in a concatenated form as $\mathbf{J}_\theta^c$, or:

$$\mathbf{J}_\theta^c = \begin{bmatrix} \mathbf{J}_{\theta 1} & . & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & . & . & . & . \\ . & . & \mathbf{J}_{\theta i} & . & . \\ . & . & . & . & . \\ . & . & . & . & \mathbf{J}_{\theta n} \end{bmatrix} . \tag{A.15}$$

Where, $\mathbf{J}_{\theta i}$ is the Jacobian of *ith* finger.

Figure A.4 shows a schematic of transformation between the spaces of twists and wrenches of the finger/object system, Payandeh, Goldenberg[36].

A.3 Summary

This appendix presented definitions of the types of contact configurations between a soft finger-tip and the object and also defined the finger/object reference coordinate frames. Models for representing the types of contact configurations between the finger-tip and an object were presented. These models were in the form of transformations between the finger-tip and contact spaces of wrenches or contact and object-contact spaces of twists, i.e. $\mathbf{H}_w$ and $\mathbf{H}_t$.

The *grasp* was modelled as a transformation between the space of wrench of the grasped object at the contact areas to the space of wrench of the object expressed in the object reference coordinate frame, i.e. the *Grasp Matrix* **G**. Another model of the *grasp* was presented as a transformation between the space of twist of the object at the contact areas to the space of twist of the object expressed in the object reference coordinate frame, i.e. the *Grasp Jacobian* **J**.

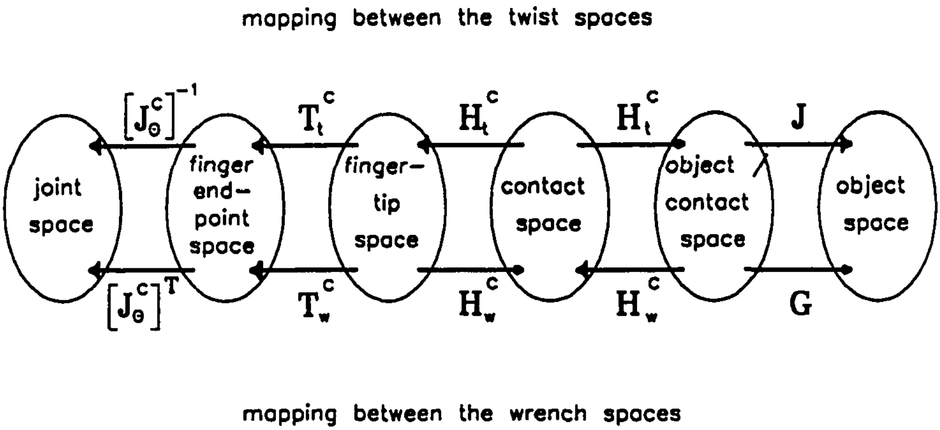


Figure A.4 – Transformations between various coordinate frames of the finger/object system.

APPENDIX B
Background Material

This appendix presents background material. This material includes some definitions, theorems and mathematical preliminaries. The Appendix is organized as follows: section B.1 gives the definition of *causality* and the *causal* representation of basic elements of a physical mechanical system; section B.2 presents a general representation of the instantaneous rigid-body displacement and the net force and moment acting on a body; section B.3 gives a definition of the principle of virtual work; section B.4 gives the solution to *a general servomechanism problem* and finally section B.5 gives the summary of this appendix.

B.1 Causality

The notion of causality was first introduce to robotic literature by Hogan[20]. This section presents a definition of *causality* and the *causal* representation of the basic elements of a mechanical system namely mass, spring and a damper .

Definition B.1: A linear system is a *causal* system if the future *output* of the system depends only on the past and current *inputs* to the system. In other words, when the *output* of a linear system at time $t=t_k$ depends only on *inputs* up to this time, then the linear system is said to be *causal*. *(Francis[37])*

Example of a *causal* element of a linear system is an *integral* ($\int$) operator. Here, the *input* u_k is a continuous (or discrete) function from $t=t_0$ to $t=t_k$ where the *output* y_k of the *integral* at $t=t_k$ is written as: Figure B.1

$$y_k = \sum_{t=t_0}^{t=t_k} u_t = \int u_t dt \quad . \tag{B.1}$$

B.1.1 Causal Representations of Physical Mechanical Elements

This subsection, based on the definition of *causality* defines the *causal* representations of the basic elements of a mechanical system namely, mass, spring and damper .

Definition B.2: In mechanical representations, *force (moment)* are defined as *through* variables because their effect are instantaneously realized through out the elements.

Definition B.3: In mechanical representations, *velocity* is defined as an *across* variable since a reference configuration of elements are required for determining their relative velocities.

For the *mass* element of a physical mechanical system, see Figure B.2, the *input* is defined as a *through* variable (force) and the *output* is defined as an *across* variable (velocity). For the *spring* element, see Figure B.3, the *input* is defined as an *across* variable (velocity) and the *output* is defined as a *through* variable (force).(MacFarlane[38])

Remark B.1 : In the above mass and spring representations, Figure B.2-B.3, the presence of the *integral* ($\int$) operator in block diagrams makes these representations *causal.*

Figure B.4 shows a representation of a *damper* element. In this Figure, velocity is defined as an *input* variable and force is defined as an *output* variable. Also, since this representation does not include an *integral* operator, the input/output relationship of a

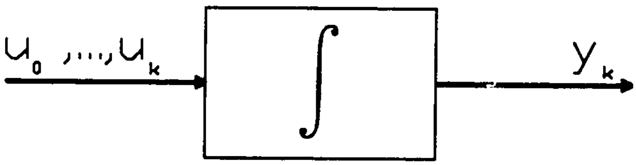


Figure B.1 – Input/output representation of an integral operator.

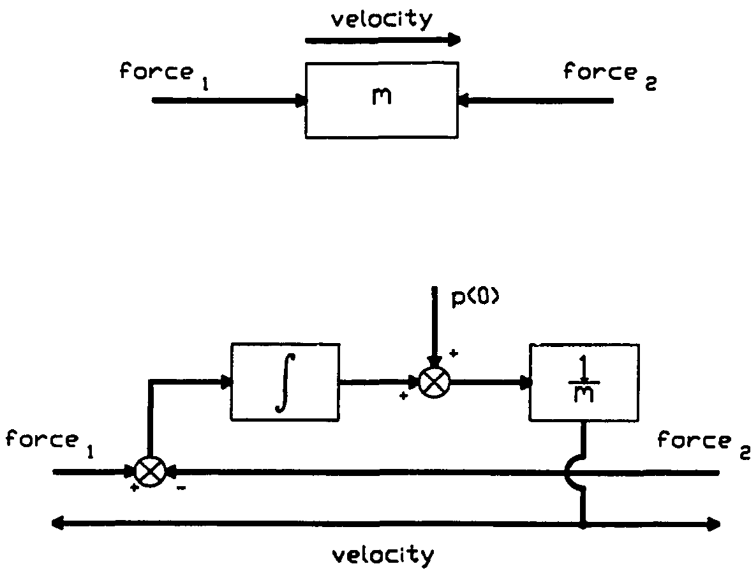


Figure B.2 – Causal representation of a mass element.

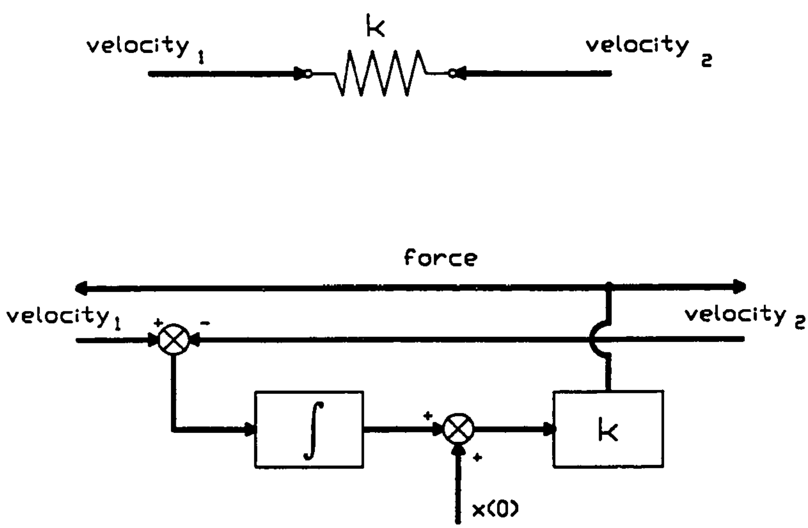


Figure B.3 – Causal representation of a spring element.

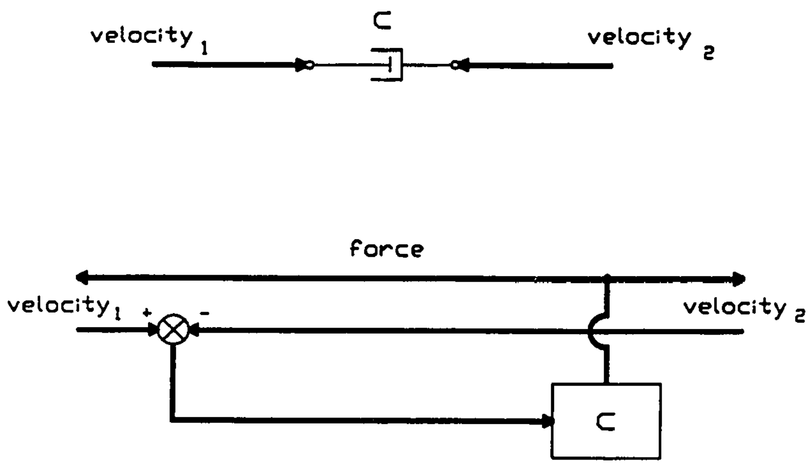


Figure B.4 – A representation of a damper element.

damper element can be reversed.

B.2 Screw, Twist and Wrench

This section presents a general method for representing a rigid body displacement and the net force and moment acting on a rigid body.

Definition B.4: A *screw* S is defined by a straight line in space known as its axis ,*i.e. screw axis,* and a scalar number ρ_s which defines its *pitch.*

A *screw* S is represented by the normalized *Plücker* line coordinates of its axis as: (Hunt[39], Payandeh and Goldenberg[40])

$$S = (s;s_o)^T = (s_1,s_2,s_3,s_4,s_5,s_6)^T = \eta_s(L,M,N,P+\rho_s L,Q+\rho_s M,R+\rho_s N)^T \quad . \quad \text{(B.2)}$$

Where η_s is defined as the intensity (magnitude) of a *screw,* L, M and N are the direction cosines of the *screw axis* and P, Q and R are the moments of this axis about a reference coordinate frame. Here, the *pitch* of the *screw* is defined as:

$$\rho_s = \frac{s_1 s_4 + s_2 s_5 + s_3 s_6}{s_1^2 + s_2^2 + s_3^2} \quad . \quad \text{(B.3)}$$

Definition B.5: Infinitesimal (virtual) displacement or velocity of a body is represented by a *twist* T about a *screw*. A *twist* is expressed as :

$$T = (\omega;v)^T = (t_1,t_2,t_3,t_4,t_5,t_6)^T = \eta_t(L,M,N,P+\rho_t L,Q+\rho_t M,R+\rho_t N)^T \quad . \quad \text{(B.4)}$$

Where η_t is the intensity of a *twist* T. The *pitch* of a *twist* is defined as:

$$\rho_t = \frac{\omega . v}{\omega . \omega} \quad . \quad \text{(B.5)}$$

When the *pitch* is finite or infinite, the intensity of a *twist* is determined as:

$$\eta_t = \|T\| = (t_1^2+t_2^2+t_3^2)^{1/2} = \|\omega\| \ .$$

$$when \quad (\rho_t{=}\infty) \quad \eta_t = \|T\| = (t_4^2+t_5^2+t_6^2)^{1/2} = \|v\| \ . \qquad \text{(B.6)}$$

Definition B.6: The net force and moment acting on a rigid-body is represented as a *wrench* about a *screw* which is expressed as :

$$W = (f\,;m_o)^T \ .$$

$$= (w_1,w_2,w_3,w_4,w_5,w_6)^T = \eta_w(L,M,N,P+\rho_w L,Q+\rho_w M,R+\rho_w N)^T \ . \qquad \text{(B.7)}$$

Where, η_w is the intensity of a *wrench*. The *pitch* of a *wrench* is defined as :

$$\rho_w = \frac{f.m_o}{f.f} \ . \qquad \text{(B.8)}$$

When the *pitch* of a *wrench* ρ_w is finite or infinite, the intensity of a *wrench* is determined as:

$$\eta_w = \|W\| = (w_1^2+w_2^2+w_3^2)^{1/2} = \|f\| \ ,$$

$$when \quad (\rho_w{=}\infty) \quad \eta_w = \|W\| = (w_4^2+w_5^2+w_6^2)^{1/2} = \|m_o\| \ . \qquad \text{(B.9)}$$

Remark B.2: When the intensity of a wrench η_w is a function of time, i.e. $\eta_w(t)=\eta_w\sin(\omega_w t)$, the wrench is referred to as a *dynamic wrench.*

Definition B.7: Two *screws* are said to be *reciprocal* to each other when the following product between the two *screws* is zero, i.e. *reciprocal* product:

$$<S_1,S_2> = <(s_1 \cdot s_{o,2})\,;\,(s_{o,1} \cdot s_2)> = 0. \qquad \text{(B.10)}$$

Where, (.) represents a dot or scalar product.

Remark B.3: The *reciprocal* product between a *wrench* and a *twist* is the rate of work that a *wrench* can produce in direction of a *twist*. In particular, when this product is zero, the *wrench* and the *twist* are *reciprocal* to each other, i.e. the rate of work done by a *wrench* W acting on a body which is free to displace about a *twist* T is zero, Ball[41].

Note B.1 : For notational convenience, a *wrench* is expressed as $W=(m_o;f)^T$. Therefore, the *reciprocal* product between a *wrench* and a *twist* is written as :

$$< W,T > = < (m_o \,.\, \omega) \,;\, (f \,.\, v) > \ ,$$

$$or \qquad < W,T > = W^T . T \ .$$

Definition B.8: The *coordinates of a twist* are the components of a vector connecting the origin of a six-dimensional space $\mathbf{R}^6$ to a point in this space specified by the coordinates of the *twist* T, i.e. $T \in \mathbf{R}^6$. The unit-bases of this space are defined as $i,j,k,\varepsilon i,\varepsilon j,\varepsilon k$.

Definition B.9: *The integral of a twist* over a period of time t is a vector defined as:

$$\int T dt = (i\int t_1 dt \ , j\int t_2 dt \ \ , k\int t_3 dt \ , \varepsilon i\int t_4 dt \ , \varepsilon j\int t_5 dt \ , \varepsilon k\int t_6 dt)^T,$$

$$or \qquad \int T dt = (\int \omega dt;\int v dt)^T \ . \qquad \text{(B.11)}$$

Definition B.10: *The derivative of a twist* is a vector defined as :

$$\frac{dT}{dt} = \left[\frac{dt_1}{dt}i \ , \ \frac{dt_2}{dt}j \ , \ \frac{dt_3}{dt}k \ , \ \frac{dt_4}{dt}\varepsilon i \ , \ \frac{dt_5}{dt}\varepsilon j \ , \ \frac{dt_6}{dt}\varepsilon k\right]^T \ ,$$

$$or \qquad \frac{dT}{dt} = \left[\frac{d}{dt}\omega;\frac{d}{dt}v\right]^T \ . \qquad \text{(B.12)}$$

Remark B.4: Based on the definition of *causality* see *definition B.1*, differentiation is not a *causal* operation because it requires the knowledge of the future input, however, this operation is approximated by *backward-difference* method, Van de Vegte[24] which is a *causal* operation.

B.3 A Definition of the Principle of Virtual Work

This section defines the virtual work principle in terms of *reciprocal product* between a *wrench* which is a vector of forces and moments and a *twist* which is a vector of virtual displacements and rotation of a rigid body.

Theorem B.1: When a mechanical system is in static equilibrium, the *reciprocal* product, see *definition B.7*, between a *wrench* acting on a body and a *twist* representation of a virtual displacement is zero.

Proof : Equilibrium represents a stationary point of the work function, i.e. potential energy function. The variation of this function at a stationary point, see Lanczos[42], with respect to the independent variables,i.e. components of a *twist* is zero, or :

$$\delta(work) = \frac{\partial(work)}{\partial t_1}\delta t_1 + ... + \frac{\partial(work)}{\partial t_6}\delta t_6 = 0 \ .$$

Where $t_1, ..., t_6$ are components of *twist* vector, virtual displacement (*definition (B.8)*). The above equation can also be written as:

$$\delta(work) = W^T . T = 0 \ , \tag{B.13}$$

Where T represents a virtual displacement and W represents a wrench. From *definition B.7* and *note B.1*, equation B.13 represents a *reciprocal* product between a *wrench* and a *twist*. □

B.4 Solution to a General Servomechanism Problem

This section presents the necessary and sufficient conditions which must be satisfied in order for a linear system to have a robust controller and also presents a general form of this controller, Davison and Goldenberg[24], Davison[26].

Consider the following state-space representation of a *linear time invariant* system :

$$\dot{x} = \mathbf{A}x+\mathbf{B}u+\mathbf{E}\omega \ .$$

$$y = \mathbf{C}x+\mathbf{D}u \ . \tag{B.14}$$

$$where \quad e = y-y_{ref} \ .$$

In the above equations, $x \in \mathbf{R}^n$ is the state vector, $u \in \mathbf{R}^m$ is the controlled input vector, $y \in \mathbf{R}^r$ is the output vector of the system which has to be regulated,i.e. $e \to 0$ for $t \to \infty$.

Definition B.11: Given a system defined by equation B.14, the system is *stabilizable* if and only if it is *controllable.* The following condition determines whether a given system is *controllable:*

$$rank\left\{\mathbf{B},\mathbf{AB},\mathbf{A}^2\mathbf{B},......,\mathbf{A}^{n-1}\mathbf{B}\right\} = n \ . \tag{B.15}$$

Definition B.12: Given a system defined by equation B.14, the system is *detectable* if and only if it is *observable.* The following condition determines whether a given system is *observable:*

$$rank\left\{\begin{matrix}\mathbf{C}\\ \mathbf{CA}\\ \mathbf{CA}^2\\ .....\\ .....\\ \mathbf{CA}^{n-1}\end{matrix}\right\} = n \ . \tag{B.16}$$

Definition B.13: Given a multi-input/multi-output (*MIMO*) system defined by equation B.14, the transmission zeros of the system are the zeros of the transfer function matrix defined to be the set of complex numbers ν which satisfies the following inequality:

$$rank \begin{bmatrix} \mathbf{A}-\nu\mathbf{I} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \le n+min(r,m) \ . \qquad \text{(B.17)}$$

Assumption B.1: It is assumed that the all *exogenous* inputs,i.e. reference inputs $y_{ref} \in \mathbf{R}^r$ and unmeasurable input disturbances $\omega \in \mathbf{R}^r$ satisfy the following linear differential equation:

$$\left[\,.\,\right]^{(p)} + \alpha_p \left[\,.\,\right]^{(p-1)} + \dots\dots\dots + \alpha_2 \left[\,\dot{}\,\right] + \alpha_1 \left[\,.\,\right] = 0 \ . \qquad \text{(B.18)}$$

Or,

$$(s^p + \alpha_p s^{p-1} + \dots\dots\dots\dots + \alpha_2 s + \alpha_1\ )\ (.) = 0 \ . \qquad \text{(B.19)}$$

Where (.) is either y_{ref} or ω .

Remark B.5: Equation B.18 models classes of unstable *exogenous* inputs,e.g. step or sinusoidal inputs

Theorem B.2: The necessary and sufficient conditions that there exists a robust controller to the linear time invariant system defined by equation B.14 such that $e \to 0$ as $t \to \infty$ for all unmeasurable input disturbances ω and reference inputs y_{ref} satisfying equation B.18 are that the following conditions all hold:

a) (**A**,**B**) be *stabilizable.*

This means that if the system has unstable modes, these modes should be affected by the controlled inputs *u*.

b) ($\mathbf{C},\mathbf{A}$) be *detectable.*

This means that if the system has unstable modes, these modes should be realized by the output y.

c) number of control inputs must be greater than or equal to the number of outputs,i.e. $m \geq r$.

d) the roots of characteristic equation defined by equation B.19 should not coincide with the transmission zeros of the system.

e) the outputs which have to be regulated must be measurable.

Proof : Davison and Goldenberg[24].□

The most general controller which regulates the system defined by equation B.14 has the following form: (see Figure B.5)

$$u = \mathbf{K}_{sta} x + \mathbf{K}_{ser} \xi \ . \qquad \text{(B.20)}$$

Where x is the state of the system, ξ is an rp vector representing the output of a general *servo-compensator* which has the following state-space model:

$$\dot{\xi} = \Lambda \xi + \beta^* e \ . \qquad \text{(B.21)}$$

Where the matrices Λ and β^* are defined as:

$$\Lambda = \tau \, block \ diag \ (\psi, \psi, \ldots\ldots, \psi) \, \tau^{-1} \ .$$

$$\text{and} \quad \beta^* = \tau \beta \ .$$

In the above τ is a non-singular real matrix, ψ is in the companion (canonical) representation of the linear differential equation representing the y_{ref} and ω, i.e. equation B.18. $\beta \in \mathbf{R}^{rp \times r}$ is a real matrix of rank r.

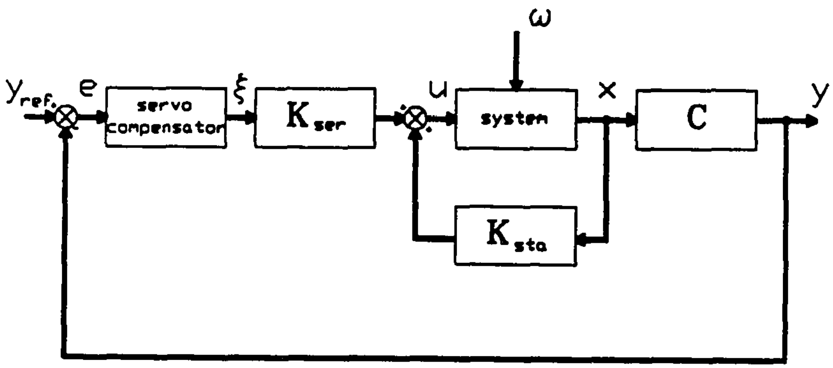


Figure B.5 - Block diagram of the general robust controller.

One form of the *servo-compensator* can be obtained by setting $\tau = \mathbf{I}$ which results in the following:

$$\Lambda = block\ diag\ (\psi, \psi, \ldots\ldots\ldots, \psi)\ . \tag{B.22}$$

and defining β^* as :

$$\beta^* = block\ diag \begin{bmatrix} \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{pmatrix} \ldots\ldots\ldots, & \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \\ 1 \end{pmatrix} \end{bmatrix} . \tag{B.23}$$

$\mathbf{K}_{sta}$ and $\mathbf{K}_{ser}$ are the gain matrices of the controller defined in equation B.23 where $\mathbf{K}_{ser}\xi$ represents the regulating part and $\mathbf{K}_{sta}x$ is the stabilizing part of the total controller u.

Remark B.6: Magnitudes of the gain matrices are obtained using standard methods, e.g. pole-placement or optimal control methods .

B.4.1 The Main Property of the Controller

The main property of the control architecture of Figure B.5 is that the asymptotic output regulation,i.e. $e \rightarrow 0$ for $t \rightarrow \infty$ occurs for any finite variations in the system parameters $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and the presence of unwanted input *exogenous* disturbances as long as the closed-loop system remains stable, i.e. the controller is *robust* .

B.5 Summary

This appendix presented some fundamental definitions and theorems which are the bases for the thesis.

Definitions of *causal* elements, see *section B.1,* of a physical mechanical system presented a method for describing the interaction between these elements. The generalized representation of a rigid-body displacement in space using the notions of *twist*, see *section B.2*, offered a unique and compact method for representing the instantaneous properties of a rigid-body. The notion of a *wrench* was also introduced to represent the generalized forces and moments acting on a rigid-body. Both of these representations are used extensively in the context of the thesis.

Section B.4 outlined the conditions for the existence and the general form of a robust controller.

APPENDIX C
Targeted Impedance

Chapter II presented a causal model of each grasping finger with soft finger-tip. This appendix outlines some methods for determining the property of the grasped object given the parameters of the model of the grasping fingers, Payandeh, Goldenberg[43]. Two general tasks are considered in this appendix; a) contact tasks and b) grasping a vibratory object.

C.1 Contact Tasks

This section first discusses the desired property of the grasped object when it makes contact with an environment. *Corollary 2.1* stated that the port of interaction between the grasped object and the environment is not a causal port. From *proposition 2.1* and *corollary 2.2* it is stated that one of the interacting bodies must have spring or damper models. Since the environment is assumed to be rigid, see *assumption A.3*, only the grasped object can have spring or damper properties. The spring and damper properties of the grasped object at the port of interaction with the environment are obtained from the spring and damper models of the grasping fingers.

Let $\mathbf{K}_{tip}$ be the combined matrix of the spring models of all the grasping fingers expressed in their corresponding finger-tips coordinate frames. The objective is to determine the spring model of the object expressed in its coordinate frame $\mathbf{K}_o$, i.e. a coordinate frame located at the port of interaction between the grasped object and the environment, as a function of $\mathbf{K}_{tip}$. For each finger, the stiffness of the the finger-tip is the resultant stiffness of two springs in series, namely, the finger spring model and the soft finger-tip, Figure 2.2a.

Theorem C.1: The spring model of the grasped object as a function of the spring models of finger-tips is given as:

$$\mathbf{K}_o = \mathbf{G}\mathbf{H}_t^c\mathbf{H}_w^c\mathbf{K}_{tip}\mathbf{H}_t^c\mathbf{H}_w^c\mathbf{G}^T \quad . \tag{C.1}$$

Proof: Let the wrench/twist relationship of the finger-tips expressed in the finger-tips coordinate frames, see Figure A.2, be given as:

$$\overline{W}_{tip} = \mathbf{K}_{tip}\overline{T}_{tip} \quad , \tag{C.2}$$

where $\overline{W}_{tip}$ and $\overline{T}_{tip}$ are the combined vector of wrenches and twists of the fingers expressed in the finger-tips reference coordinate frames. From *remark A.5,* these vectors can be expressed in the contact reference coordinate frames as:

$$[\mathbf{H}_w^c]^{-1}\overline{W}_c = \mathbf{K}_{tip}\mathbf{H}_t^c\overline{T}_c \quad , \tag{C.3}$$

or,

$$\overline{W}_c = \mathbf{H}_w^c\mathbf{K}_{tip}\mathbf{H}_t^c\overline{T}_c \quad .$$

Similarly, $\overline{W}_c$ and $\overline{T}_c$ can be expressed in the object/contact reference coordinate frames as:

$$\mathbf{H}_w^c\overline{W}_{oc} = \mathbf{H}_w^c\mathbf{K}_{tip}\mathbf{H}_t^c[\mathbf{H}_t^c]^{-1}\overline{T}_{oc} \quad , \tag{C.4}$$

or the above can be written as:

$$\overline{W}_{oc} = [\mathbf{H}_w^c]^{-1}\mathbf{H}_w^c\mathbf{K}_{tip}\mathbf{H}_t^c[\mathbf{H}_t^c]^{-1}\overline{T}_{oc} \quad , \tag{C.5}$$

replacing $[\mathbf{H}_w^c]^{-1}$ and $[\mathbf{H}_t^c]^{-1}$ with their corresponding right and left generalized inverses, see *remark A.4,* we have:

$$\overline{W}_{oc} = \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{K}_{tip} \mathbf{H}_t^c \mathbf{H}_w^c \overline{T}_{oc} \quad . \tag{C.6}$$

From *definition A.14* and *definition A.15*, expressing $\overline{W}_{oc}$ and $\overline{T}_{oc}$ in the object reference coordinate frame, we can write the following:

$$\mathbf{G}^{-1} W_o = \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{K}_{tip} \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{J}^{-1} T_o \quad , \tag{C.7}$$

from the results of *theorem A.2*, the above equation can be written as:

$$W_o = \mathbf{G} \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{K}_{tip} \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{G}^T T_o \quad . \tag{C.8}$$

The above equation is the wrench/twist relationship of the grasped object expressed in its reference coordinate frame as a function of the stiffness of the finger-tips is written as:

$$\mathbf{K}_o = \mathbf{G} \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{K}_{tip} \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{G}^T \quad . \quad \square \tag{C.9}$$

Corollary C.1: Similar derivation can be followed to determine the damper model of the grasped object $\mathbf{C}_o$ as a function of the damper models of each finger expressed in their corresponding finger-tips reference coordinates $\mathbf{C}_{tip}$, or:

$$\mathbf{C}_o = \mathbf{G} \mathbf{H}_t^c \mathbf{H}_w^c \mathbf{C}_{tip} \mathbf{H}_t^c \mathbf{H}_w^t \mathbf{G}^T \quad . \tag{C.10}$$

The spring and damper models of each finger which can result in the desired spring and damper model of the grasped object with respect to its reference coordinate frame is referred to the *targeted stiffness* and *targeted damping*. In general, equation C.9 and C.10 can be used as constraint relationships in an optimization algorithm for determining a suitable combinations of the stiffness and damping matrices of fingers, Shimoga[44].

C.2 Grasping a Vibratory Object

When the grasped object vibrates, the objective of the grasping fingers must be such that they can reduce the vibration to zero. Examples are holding the dentist drill or the barbers electric hair-clipper.

In order to reduce the vibration to zero, the objective is to write the equations of motion of groups of fingers in the direction where they can push on the object. These equations are obtained as if the object is glued to these fingers. Then, the objective becomes to determine conditions between the model parameters of the fingers such that the vibration of the grasped object is reduced to zero.

Figure C.1 shows a schematic where the object is vibrating in the x-direction. Fingers #1 and #2 together can oppose the motion of the object by pushing similar to fingers #3 and #4. The equations of motions of the finger #1 and #2 and the object can be written as:

$$\begin{bmatrix} m_o s^2+(c_{u1,x}+c_{u2,x})s+(k_{u1,x}+k_{u2,x}) & -c_{u1,x}s-k_{u1,x} & -c_{u2,x}s-k_{u2,x} \\ -c_{u1,x}s-k_{u1,x} & -m_{fe1,x}s^2+(c_{fe1,x}+c_{u1,x})s+(k_{fe1,x}+k_{u1,x}) & 0 \\ -c_{u2,x}s-k_{u2,x} & 0 & -m_{fe}s^2+(c_{fe2,x}+c_{u2,x})s+(k_{fe2,x}+k_{u2,x}) \end{bmatrix} \begin{Bmatrix} \bar{x}_{o,x} \\ \bar{x}_{fe1,x} \\ \bar{x}_{fe2,x} \end{Bmatrix} = \begin{Bmatrix} \bar{f}_{o,x} \\ 0 \\ 0 \end{Bmatrix} . \tag{C.11}$$

Solving for the amplitude of the vibration of the grasped object $\bar{x}_{o,x}$ and determining the condition such that this amplitude is equal to zero results in the following constraint equation:

$$A^2+B^2=0. , \tag{C.12}$$

where:

$$A=m_{fe1,x}m_{fe2,x}\omega_t^4-(m_{fe1,x}(k_{fe2,x}+k_{u2,x})-(c_{fe1,x}+c_{u2,x})(c_{fe2,x}+c_{u2,x})$$
$$-m_{fe2,x}(k_{fe1,x}+k_{u1,x})\omega_t^2+(k_{fe1,x}+k_{u1,x})(k_{fe2,x}+k_{u2,x}) . \tag{C.13}$$

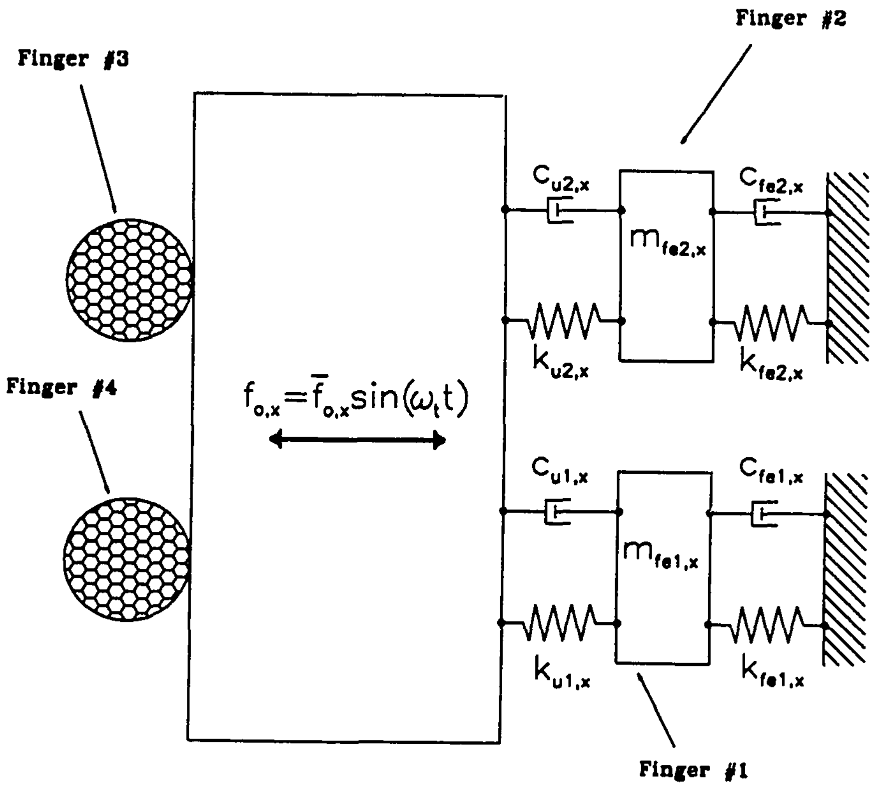


Figure C.1 – Fingers grasping a vibratory object.

$$B=(-m_{fe\,1,x}(c_{fe\,2,x}+c_{u\,2,x})-m_{fe\,2,x}(c_{fe\,1,x}+c_{u\,1,x}))\omega_t^3+((c_{fe\,1,x}+c_{u\,1,x})(k_{fe\,2,x}+k_{u\,2,x})+$$

$$(k_{fe\,1,x}+k_{u\,1,x})(c_{fe\,2,x}+c_{u\,2,x}))\omega_t \qquad \text{(C.14)}$$

Selecting the model parameters of the fingers #1,#2 such that they satisfy the constraint relationship of equation C.11 can result in the vibration of the grasped object approaches zero. However, since these fingers can not pull the grasped object, the assumption is by selecting the model parameters of finger #3 and #4 such that they satisfy a similar constraint relationship of equation C.11, their cooperative response can reduce the vibration of the object to zero. The proof of the effectiveness of this approach is subject to experimental verification which is beyond the scope of the thesis.

In this thesis it is assumed that the model parameters of each grasping finger are given, i.e. *targeted impedance* of the finger $\mathbf{Z}_t$, and the objective of the thesis is to match the actual impedance of the finger with the desired one.

In general, the impedance model represents a relationship between the force and displacement, Hogan[20].

APPENDIX D
Dynamic Model of a 2DOF Finger

This appendix presents a linear dynamic model of a *2DOF* finger of a *dexterous mechanical hand*.

Let the nonlinear dynamic model of a *2DOF* finger be given as: (Fu, Gonzales and Lee[45])

$$W_\theta = \mathbf{M}(\theta)\ddot{\theta} + \mathbf{N}(\theta,\dot{\theta}) + \mathbf{G}(\theta) \ , \tag{D.1}$$

where, $W_\theta = (\tau_{\theta 1}, \tau_{\theta 2})^T$ is a vector of joint torques expressed in the finger configuration coordinate frame, $\mathbf{M}(\theta)$ is the inertia matrix of a finger expressed in its configuration coordinate frame, $\mathbf{N}(\theta,\dot{\theta})$ is the matrix containing the nonlinear terms associated with the Coriolis and centrifugal terms and $\mathbf{G}(\theta)$ is the term associated with the gravity.

The perturbed model of the dynamics equation of a finger about an equilibrium configuration is written as:

$$W_\theta + \delta W_\theta = \mathbf{M}(\theta+\delta\theta)(\ddot{\theta}+\delta\ddot{\theta}) + \mathbf{N}(\theta+\delta\theta,\dot{\theta}+\delta\dot{\theta}) + \mathbf{G}(\theta+\delta\theta) \ . \tag{D.2}$$

Assuming that $\mathbf{M}(\theta+\delta\theta) = \mathbf{M}(\theta)$ and using *Taylor Series Expansion* to expand terms $\mathbf{N}$ and $\mathbf{G}$ we have : (Davison[26])

$$\mathbf{N}(\theta+\delta\theta,\dot{\theta}+\delta\dot{\theta}) = \mathbf{N}(\theta,\dot{\theta}) + \left[\frac{\partial \mathbf{N}(\theta,\dot{\theta})}{\partial\theta}\right]_o \delta\theta + \left[\frac{\partial \mathbf{N}(\theta,\dot{\theta})}{\partial\dot{\theta}}\right]_o \delta\dot{\theta} + \ldots\ldots\ldots\ldots \tag{D.3}$$

$$\mathbf{G}(\theta+\delta\theta) = \mathbf{G}(\theta) + \left[\frac{\partial \mathbf{G}(\theta)}{\partial\theta}\right]_o \delta\theta + \ldots\ldots\ldots\ldots \tag{D.4}$$

Ignoring the second and higher order terms in $\delta\theta$ and $\delta\dot{\theta}$ in the above expansions, equation D.2 is written as:

$$W(\theta)+\delta W(\theta) = \mathbf{M}(\theta)+\mathbf{N}(\theta,\dot{\theta})+\mathbf{N}_1(\delta\theta)+\mathbf{N}_2(\delta\dot{\theta})+\mathbf{G}(\theta)+\mathbf{G}_1(\delta\theta) \ . \tag{D.5}$$

Substituting equation D.1 into equation D.5 for W_θ we obtained the following equation:

$$\delta W(\theta)=\mathbf{M}(\theta)\delta\ddot{\theta}+\mathbf{N}_2\delta\dot{\theta}+(\mathbf{N}_1+\mathbf{G}_1)\delta\theta \ , \tag{D.6}$$

where for example, for a *2DOF* finger having cylindrical links, the terms $\mathbf{M}(\theta)$,$\mathbf{N}_1$,$\mathbf{N}_2$, and $\mathbf{G}_1$ are written as :

$$\mathbf{M}(\theta) = \begin{bmatrix} 1/3m_1l_1^2+1/3m_2l_2^2+m_2l_1^2+m_2l_1l_2c_1 & 1/3m_2l_2^2+1/2m_2l_1l_2c_2 \\ 1/3m_2l_2^2+1/2m_2l_1l_2c_2 & 1/3m_2l_2^2 \end{bmatrix} ,$$

$$\mathbf{N}_1 = \begin{bmatrix} -m_2l_1l_2\dot{\theta}_2s_2 & -m_2l_1l_2(\dot{\theta}_1+\dot{\theta}_2)s_2 \\ m_2l_1l_2\dot{\theta}_1s_2 & 0 \end{bmatrix} ,$$

$$\mathbf{N}_2 = \begin{bmatrix} 0 & -m_2l_1l_2(\dot{\theta}_1\dot{\theta}_2+\dot{\theta}_2/2)c_2 \\ 0 & 1/2m_2l_1l_2\dot{\theta}_1c_2 \end{bmatrix} ,$$

$$\mathbf{G}_1 = \begin{bmatrix} (-1/2m_1gl_1+m_2gl_1)s_1-1/2m_2gl_2s_{12} & -1/2m_2gl_2s_{12} \\ -1/2m_2gl_2s_{12} & -1/2m_2gl_2s_{12} \end{bmatrix} ,$$

where: $c_1=cos(\theta_1)$,$c_2=cos(\theta_2)$, $s_1=\sin(\theta_1)$, $s_2=\sin(\theta_2)$, $s_{12}=\sin(\theta_1+\theta_2)$,$l_1$= length of link 1 , l_2= length of link 2 , m_1= mass of link 1, m_2= mass of link 2 and g= gravity acceleration. Rewriting equation D.6 by setting:

$$\mathbf{M}_\theta = \mathbf{M}(\theta) \ ; \ \mathbf{C}_\theta = \mathbf{N}_1 \ ; \ \mathbf{K}_\theta = (\mathbf{N}_2+\mathbf{G}_1) \ .$$

Also, defining a vector T_θ as a vector of joint velocities expressed in the finger configuration coordinates as $T_\theta=(\delta\dot{\theta}_1,\delta\dot{\theta}_2)^T$, the linearized dynamic model of a finger is expressed as:

$$\mathbf{M}_\theta \dot{T}_\theta + \mathbf{C}_\theta T_\theta + \mathbf{K}_\theta \int T_\theta dt = W_\theta \quad . \qquad \text{(D.7)}$$

Remark D.1: When a finger of a *dexterous mechanical hand* makes contact with an object, the coordinates of the initial contact of the finger-tip with an object can be used as an operating point where the nonlinear model of the finger is linearized.

The linearized dynamic model of a finger can also be expressed in a coordinate frame located in the finger end-point as:

$$W_{act} = \mathbf{M}_{fe} \dot{T}_{fe} + \mathbf{C}_{fe} T_{fe} + \mathbf{K}_{fe} \int T_{fe} dt \quad , \qquad \text{(D.8)}$$

or,

$$W_{act} = \frac{1}{s} \mathbf{Z}_f T_{fe} \quad ,$$

where,

$$\mathbf{Z}_f = \mathbf{M}_{fe} s^2 + \mathbf{C}_{fe} s + \mathbf{K}_{fe}$$

where, the parameters of the linear dynamic equation of the finger expressed in the finger end-point reference coordinate frame in terms of the parameters of the equation (D.7) are defined as: (Khatib[46])

$$\mathbf{M}_{fe} = \mathbf{J}_\theta^{-T} \mathbf{M}_\theta \mathbf{J}_\theta^{-1} \quad ,$$

$$\mathbf{C}_{fe} = \mathbf{J}_\theta^{-T} (-\mathbf{M}_\theta \mathbf{J}_\theta^{-1} \dot{\mathbf{J}}_\theta \mathbf{J}_\theta^{-1} + \mathbf{C}_\theta \mathbf{J}_\theta^{-1}) \quad ,$$

$$\mathbf{K}_{fe} = \mathbf{J}_\theta^{-T} \mathbf{K}_\theta \quad ,$$

$$W_{act} = \mathbf{J}_\theta^{-T} W_\theta \quad ,$$

where, $\mathbf{J}_\theta$ is a Jacobian of a finger, *definition A.17.*

Remark D.2: When a finger is grasping an object and the plane of a finger is horizontal, the dynamics equation of a finger defined in equation D.8 is simplified as:

$$W_{act} = \mathbf{M}_{fe}\dot{T}_{fe} \quad . \tag{D.9}$$

Remark D.3: The nonlinear dynamic equation, see equation D.1, expressed in the finger end-point can also be written as:

$$W_{act} = \mathbf{Q}\dot{T}_{fe} + f(T_{fe}, \int T_{fe}dt) \quad , \tag{D.10}$$

where, **Q** represents the nonlinear inertia matrix and $f\,(T_{fe}, \int T_{fe}dt)$ represents the Coriolis and centrifugal components.

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